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A New Soft Set Operation: Complementary Soft Binary Piecewise Theta (θ) Operation

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ABSTRACT

Soft set theory, introduced by Molodtsov, is an efficacious mathematical tool to deal with uncertainty and it has been applied to many fields both as a theoretical and application aspect. Since its inception, different kinds of soft set operations are defined and used in various types. In this paper, we define a new kind of soft set operation called, complementary soft binary piecewise theta operation and we investigate its basic algebraic properties. Moreover, it is aimed to contribute to the soft set literature by examining the relationships between this new soft set operation and some other types of soft set operations by examing the distribution of complementary soft binary piecewise theta operation over extended soft set operations, complementary extended soft set operations, soft binary piecewise operations, complementary soft binary piecewise operations and restricted soft set operations.

Yeni Bir Esnek Küme İşlemi: Tümleyenli Esnek İkili Parçalı Theta (θ) İşlemi

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ÖZ

Molodtsov tarafından ortaya atılan esnek küme teorisi, belirsizlikle başa çıkmak için etkili bir matematiksel araçtır ve hem teorik hem de uygulama yönüyle birçok alana uygulanmıştır. Başlangıcından bu yana, farklı çeşitlerde esnek küme işlemleri tanımlanmış ve çeşitli türlerde kullanılmıştır. Bu çalışmada, tümleyenli esnek ikili parçalı teta işlemi adı verilen yeni bir esnek küme işlemi tanımlanmış ve temel cebirsel özellikleri araştırılmıştır. Ayrıca tümleyenli esnek ikili parçalı teta işleminin genişletilmiş esnek küme işlemleri, tümleyenli genişletilmiş esnek küme işlemleri, esnek ikili parçalı işlemler, tümleyenli esnek ikili parçalı işlemler ve kısıtlanmış esnek küme işemleri üzerine dağılması incelenerek bu yeni esnek küme işlemi ile diğer esnek küme işlemleri arasındaki ilişkiler elde edilerek esnek küme literatürüne katkı sağlanması amaçlanmaktadır.

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1. Introduction

The existence of some types of uncertainty in the problems of many fields such as economics, environmental and health sciences, engineering prevents us from using classical methods to solve the problems successfully. There are three well-known basic theories that we can consider as a mathematical tool to deal with uncertainties, which are Probability Theory, Fuzzy Set Theory and Interval Mathematics. But since all these theories have their own shortcomings, Molodtsov (1999) introduced Soft Set Theory as a mathematical tool to overcome these uncertainties. Since then, this theory has been applied to many fields including information systems, decision making, optimization theory, game theory, operations research, measurement theory and so on. Studies on fuzzy modeling such as Linear Diophantine Fuzzy Sets (Riaz and Hashimi, 2019; Ayub et al., 2021), Linear Diophantine Fuzzy aggregation operators (Riaz et al., 2023), Spherical Linear Diophantine Fuzzy Sets (Riaz et al., 2021) etc. are some top recent topics as novel mathematical approachs to model vagueness and uncertainty in decision-making problems. Maji et al. (2003) and Pei and Miao (2005) made the first contributions as regards soft set operations. After then, several soft set operations (restricted and extended soft set operations) were introduced and examined by Ali et al. (2009). Sezgin and Atagün (2011) illustrated the basic properties of soft set operations and discussed and the interconnections of soft set operations with each other. They also defined the notion of restricted symmetric difference of soft sets and investigated its properties. A new soft set operation called extended difference of soft sets was defined by Sezgin et al. (2019), and Stojanovic (2021) defined extended symmetric difference of soft sets and investigated its properties. When the studies are examined, we see that the operations in soft set theory proceed under two main headings, as restricted soft set operations and extended soft set operations.

Çağman (2021) proposed two conditional complements of sets as a new concept of set theory, i.e., inclusive complement and exclusive complement and explored the relationships between them. By the inspiration of this study, Sezgin et al. (2023c) introduced some new complements of sets. Aybek (2024) transferred these complements to soft set theory, and some new restricted soft set operations and extended soft set operations was defined. Demirci (2024), Sarıalioğlu (2024), Akbulut (2024) defined a new type of extended operation by changing the form of extended soft set operations using the complement at the first and second row of the piecewise function of extended soft set operations and studied the basic properties of them in detail. Moreover, a new type of soft difference operations was introduce by Eren (2019) and by being inspired this study, Yavuz (2024) and Sezgin and Yavuz (2023a) defined some new soft set operations, which is called soft binary piecewise operations and their basic properties were studied in detail, too. Also, Sezgin and Demirci (2023), Sezgin and Atagün (2023), Sezgin and Yavuz (2023b), Sezgin and Aybek, 2023; Sezgin et al. (2023a), Sezgin et al. (2023b), Sezgin and Dagtoros (2023) continued their work on soft set operations by defining a new type of soft binary piecewise operation. They changed the form of soft binary piecewise operation by using the complement at the first row of the soft binary piecewise operations.

The aim of this study is to contribute to the literature of soft set theory by describing a new soft set operation which we call "complementary soft binary piecewise theta operation". For this purpose, definition of the operation and its example are given, the algebraic properties, such as closure, association, unit and inverse element and abelian property of this new operation are examined in detail. It is aimed to contribute to the soft set literature by obtaining the distributions of the complementary soft binary piecewise theta operation over extended soft set operations, complementary extended soft set operations, soft binary piecewise operations, complementary soft binary piecewise operations and restricted soft set operations.

2. Preliminaries

In this section, some basic concepts related to soft set theory are compiled and given.

Definition 2.1. Let U be the universal set, E be the parameter set, $\mathrm{P(U)}$ be the power set of U and $\mathrm{D} \subseteq \mathrm{E}$. A pair (F, D) is called a soft set over U where F is a set-valued function such that $\mathrm{F}\colon \mathrm{D} \to \mathrm{P(U)}$. (Molodtsov, 1999)

The set of all the soft sets over U is designated by $\mathrm{S_E(U)}$, and throughout this paper, all the soft sets are the elements of $\mathrm{S_E(U)}$.

Definition 2.2. (Z, D) is called a relative null soft set (with respect to the parameter set D), denoted by $\emptyset_\mathrm{D}$, if $\mathrm{Z(t)} = \emptyset$ for all $\mathrm{t} \in \mathrm{D}$ and (Z, D) is called a relative whole soft set (with respect to the parameter set D), denoted by $\mathrm{U_D}$ if $\mathrm{Z(t)} = \mathrm{U}$ for all $\mathrm{t} \in \mathrm{D}$. The relative whole soft set $\mathrm{U_E}$ with respect to the universe set of parameters E is called the absolute soft set over U (Ali et.al., 2009)

Definition 2.3. For two soft sets (Z, D) and (R, J), we say that (Z, D) is a soft subset of (R, J) and it is denoted by $(\mathrm{Z}, \mathrm{D}) \tilde{\subseteq} (\mathrm{R}, \mathrm{J})$, if $\mathrm{D} \subseteq \mathrm{J}$ and $\mathrm{Z(t)} \subseteq \mathrm{R(t)}$, $\forall \mathrm{t} \in \mathrm{D}$. Two soft sets (Z, D) and (R, J) are said to be soft equal if (Z, D) is a soft subset of (R, J) and (R, J) is a soft subset of (Z, D) (Pei and Miao, 2005).

Definition 2.4. The relative complement of a soft set (Z, D), denoted by $(\mathrm{Z}, \mathrm{D})^\mathrm{r}$, is defined by $(\mathrm{Z}, \mathrm{D})^\mathrm{r} = (\mathrm{Z^r}, \mathrm{D})$, where $\mathrm{Z^r}\colon \mathrm{D} \to \mathrm{Z(U)}$ is a mapping given by $(\mathrm{Z}, \mathrm{D})^\mathrm{r} = \mathrm{U} \setminus \mathrm{Z(t)}$ for all $\mathrm{t} \in \mathrm{D}$

(Ali et al., 2009). From now on, $U\setminus Z(t)=[Z(t)]'$ will be designated by Z'(t) for the sake of designation.

Çağman (2021) introduced two conditional complements of sets as a new concept of set theory, that is, inclusive complement and exclusive complement. For the ease of illustration, we show these complements as + and θ, respectively. These complements are binary operations and are defined as follows: Let D and J be two subsets of U. J-inclusive complement of D is defined by, D+J=D'∪J and J-Exlusive complement of D is defined by Dθ J =D'∩J'. Here, U refers to a universe, D' is the complement of D over U. For more information, we refer to Çağman (2021). Sezgin et al. (2023c) examined the relations between these two complements in detail and they also introduced such new three complements as binary operations of sets as follows: Let D and J be two subsets of U. Then, D*J=D'∪J', DγJ=D'∩J, D$\boldsymbol{\lambda}$J=D∪J' (Sezgin et al., 2023c). Aybek (2024) conveyed these set operations to soft sets, and defined restricted, extended soft set operations, also examined their properties. As a summary for soft set operations, we can categorize all types of soft set operations as follows: Let "∇" be used to represent the set operations (i.e., here ∇ can be ∩,∪,\, Δ, +,θ, *, λ,γ), then restricted operations, extended operations, complementary extended operations, soft binary piecewise operations, complementary soft binary piecewise operations are defined in soft set theory as follows:

Definition 2.5. Let (Z,D) and (R,J) be soft sets over U. The restricted ∇ operation of (Z,D) and (R,J) is the soft set (S,F), denoted by $(Z,D)\nabla_R(R,J)=(S,F)$, where $F=D\cap J\neq\emptyset$ and $\forall t\in F$, $S(t)=Z(t)\nabla R(t)$. (Ali et al., 2009; Sezgin and Atagün, 2011; Aybek, 2024)

Definition 2.6. Let (Z,D) and (R,J) be soft sets over U. The extended ∇ operation of (Z,D) and (R,J) is the soft set (S,F), denoted by $(Z,D)\nabla_\varepsilon(R,J)=(S,F)$, where $F=D\cup J$ and $\forall t\in F$,

$$S(t)=\begin{cases} Z(t), & t\in D\setminus J,\\ R(t), & t\in J\setminus D,\\ Z(t)\nabla R(t), & t\in D\cap J.\end{cases}$$

(Maji et al., 2003; Ali et al., 2009; Sezgin et al., 2019; Stojanovic, 2021; Aybek, 2024)

Definition 2.7. Let (Z,D) and (R,J) be soft sets over U. The complementary extended ∇ operation of (Z,D) and (R,J) is the soft set (S,F), denoted by, $(Z,D)\overset{*}{\nabla}_\varepsilon(R,J)=(S,F)$, where $F=D\cup J$ and $\forall t\in F$,

$$S(t)=\begin{cases} Z'(t), & t\in D\setminus J,\\ R'(t), & t\in J\setminus D,\\ Z(t)\nabla R(t), & t\in D\cap J.\end{cases}$$

(Sarıalioğlu, 2024; Demirci, 2024; Akbulut, 2024)

Definition 2.8. Let (Z, D) and (R, J) be soft sets over U. The soft binary piecewise ∇ operation of (Z, D) and (R, J) is the soft set (S,D), denoted by, $(P, D)\underset{\nabla}{\sim}(R, J) = (S, D)$, where $\forall t \in D$,

$$S(t)=\begin{cases} Z(t), & t\in D\setminus J \\ Z(t)\nabla R(t), & t\in D\cap J \end{cases}$$

(Eren, 2019; Yavuz, 2024, Sezgin and Yavuz, 2023a)

Definition 2.9. Let (Z, D) and (R, J) be soft sets over U. The complementary soft binary piecewise ∇ operation of (Z, D) and (R, J) is the soft set (S,D), denoted by, $(P, D) \underset{\nabla}{\overset{*}{\sim}} (R, J) = (S, D)$, where $\forall t \in D$;

$$S(t)=\begin{cases} Z'(t), & t\in D\setminus J \\ Z(t)\nabla R(t), & t\in D\cap J \end{cases}$$

(Sezgin and Demirci, 2023; Sezgin and Atagün, 2023; Sezgin and Aybek, 2023; Sezgin et al., 2023a, Sezgin et al., 2023b; Sezgin and Yavuz, 2023b; Sezgin and Dagtoros, 2023)

3. Complementary Soft Binary Piecewise Theta (θ) Operation And Its Properties

Definition 3.1. Let (Z, D) and (R, J) be soft sets over U. The complementary soft binary piecewise theta (θ) operation of (Z, D) and (R, J) is the soft set (S,D), denoted by, $(Z, D) \underset{\theta}{\overset{*}{\sim}} (R, J) = (S, D)$, where $\forall t \in D$,

$$S(t)=\begin{cases} Z'(t), & t\in D\setminus J \\ Z'(t)\cap R'(t), & t\in D\cap J \end{cases}$$

Example 3.2. Let $E=\{e_1,e_2,e_3,e_4,e_5\}$ be the parameter set $D=\{e_1, e_3, e_5\}$ and $J=\{e_1, e_2, e_4\}$ be the subsets of E and $U=\{h_1,h_2,h_3,h_4,h_5\}$ be the initial universe set. Assume that (Z,D) and (R,J) are the soft sets over U defined as follows: $(Z,D)=\{(e_1,\{h_1,h_3\}), (e_3,\{h_2,h_4\}), (e_5,\{h_2,h_4, h_5\})\}$. $(R,J)=\{(e_1,\{h_1,h_4\}), (e_2,\{h_2,h_3\}),(e_4,\{h_4, h_5\})\}$. Let $(Z,D) \underset{\theta}{\overset{*}{\sim}} (R,J)=(S,D)$. Then,

$$S(t)=\begin{cases} Z'(t), & t\in D\backslash J \\ Z'(t)\cap R'(t), & t\in D\cap J \end{cases}$$

$D=\{e_1, e_3, e_5\}$ and $D\backslash J=\{e_3, e_5\}$, so $S(e_3)=Z'(e_3)=\{h_1,h_3,h_5\}$, $S(e_5)=Z'(e_5)=\{h_1,h_3\}$. And since $D\cap J=\{e_1\}$ so $S(e_1)=Z'(e_1)\cap R'(e_1)=\{h_2,h_4,h_5\}\cap\{h_2,h_3,h_5\}=\{h_2,h_5\}$. Thus, $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)=\{(e_1,\{h_2,h_5\}), (e_3,\{h_1,h_3,h_5\}), (e_5,\{h_1,h_3\})\}$.

Theorem 3.3. (Algebraic properties of the operation)

1) The set $S_E(U)$ is closed under the operation $\underset{\theta}{\overset{*}{\sim}}$. That is, when (Z,D) and (R,J) are two soft sets over U, then so is $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)$.

2) $[(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,D)]\underset{\theta}{\overset{*}{\sim}}(S,D)\neq(Z,D)\underset{\theta}{\overset{*}{\sim}}[(R,D)\underset{\theta}{\overset{*}{\sim}}(S,D)]$

Proof: Let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,D)=(T,D)$, where $\forall t\in D$;

$$T(t)=\begin{cases} Z'(t), & t\in D\backslash D=\emptyset \\ Z'(t)\cap R'(t), & t\in D\cap D=D \end{cases}$$

Let $(T,D)\underset{\theta}{\overset{*}{\sim}}(S,D)=(M,D)$, where $\forall t\in D$;

$$M(t)=\begin{cases} T'(t), & t\in D\backslash D=\emptyset \\ T'(t)\cap S'(t), & t\in D\cap D=D \end{cases}$$

Thus,

$$M(t)=\begin{cases} T'(t), & t\in D\backslash D=\emptyset \\ [Z(t)\cup R(t)]\cap S'(t), & t\in D\cap D=D \end{cases}$$

Let $(R,D)\underset{\theta}{\overset{*}{\sim}}(S,D)=(L,D)$, where $\forall t\in D$;

$$L(t)=\begin{cases} R'(t), & t\in D\setminus D=\emptyset \\ R'(t)\cap S'(t), & t\in D\cap D=D \end{cases}$$

Let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(L,D)=(N,D)$, where $\forall t\in D$;

$$N(t)=\begin{cases} Z'(t), & t\in D\setminus D=\emptyset \\ Z'(t)\cap L'(t), & t\in D\cap D=D \end{cases}$$

Thus,

$$N(t)=\begin{cases} Z'(t), & t\in D\setminus D=\emptyset \\ Z'(t)\cap [R(t)\cup S(t)], & t\in D\cap D=D \end{cases}$$

It is seen that $M\neq N$.

That is, for the soft sets whose parameter set are the same, the operation $\underset{\theta}{\overset{*}{\sim}}$ has not associativity property on the set $S_E(U)$. Moreover, we have the following:

3) $[(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)]\underset{\theta}{\overset{*}{\sim}}(S,F)\neq (Z,D)\underset{\theta}{\overset{*}{\sim}}[(R,J)\underset{\theta}{\overset{*}{\sim}}(S,F)]$.

Proof: Let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)=(T,D)$, where $\forall t\in D$;

$$T(t)=\begin{cases} Z'(t), & t\in D\setminus J \\ Z'(t)\cap R'(t), & t\in D\cap J \end{cases}$$

Let $(T,D)\underset{\theta}{\overset{*}{\sim}}(S,F)=(M,D)$, where $\forall t\in D$;

$$M(t)=\begin{cases} T'(t), & t\in D\setminus F \\ T'(t)\cap S'(t), & t\in D\cap F \end{cases}$$

Thus,

$$M(t)=\begin{cases} Z(t), & t\in(D\backslash J)\backslash F=D\cap J'\cap F' \\ Z(t)\cup R(t), & t\in(D\cap J)\backslash F=D\cap J\cap F' \\ Z(t)\cap S'(t), & t\in(D\backslash J)\cap F=D\cap J'\cap F \\ [\,Z(t)\cup R(t)]\cap S'(t), & t\in(D\cap J)\cap F=D\cap J\cap F \end{cases}$$

Let $(R,J)\underset{\theta}{\overset{*}{\sim}}(S,F)=(K,J)$, where $\forall t\in J$;

$$K(t)=\begin{cases} R'(t), & t\in J\backslash F \\ R'(t)\cap S'(t), & t\in J\cap F \end{cases}$$

Let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(K,J)=(Y,D)$, where $\forall t\in D$;

$$Y(t)=\begin{cases} Z'(t), & t\in D\backslash J \\ Z'(t)\cap K'(t) & t\in D\cap J \end{cases}$$

Thus,

$$Y(t)=\begin{cases} Z'(t), & t\in D\backslash J \\ Z'(t)\cap R(t), & t\in D\cap(J\backslash F)=D\cap J\cap F' \\ Z'(t)\cap[R(t)\cup S(t)], & t\in D\cap(J\cap F)=D\cap J\cap F \end{cases}$$

He re let's handle $t\in D\backslash J$ in the second equation of the first line. Since $D\backslash J=D\cap J'$, if $t\in J'$, then $t\in F\backslash J$ or $t\in(J\cup F)'$. Hence, if $t\in D\backslash J$, then $t\in D\cap J'\cap F'$ or $t\in D\cap J'\cap F$. Thus, it is seen that M=Y. That is, for the soft sets whose parameter set are not the same, the operation $\underset{\theta}{\overset{*}{\sim}}$ has not associativity property on the set $S_E(U)$.

4) $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)\neq(R,J)\underset{\theta}{\overset{*}{\sim}}(Z,D)$.

Proof : While the parameter set of the soft set of the left hand side is D; the parameter set of the soft set of the right hand side is J. Thus, by the definition of soft equality, the operation $\underset{\theta}{\overset{*}{\sim}}$ has not commutative property in the set $S_E(U)$, where the parameter sets of the soft sets are different. However it is easy to see that

$$(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,D)=(R,D)\underset{\theta}{\overset{*}{\sim}}(Z,D).$$

That is to say, the operation $\underset{\theta}{\overset{*}{\sim}}$ has commutative property in the set $S_E(U)$, where the parameter sets of the soft sets are the same.

5) $(Z,D)\underset{\theta}{\overset{*}{\sim}}(Z,D)=(Z,D)^r$.

Proof: Let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(Z,D)=(S,D)$. Then, $\forall t\in D$;

$$S(t)=\begin{cases} Z'(t), & t\in D\setminus D=\emptyset \\ Z'(t)\cap Z'(t), & t\in D\cap D=D \end{cases}$$

Here, $\forall t\in D$, $S(t)=Z'(t)\cap Z'(t)=Z'(t)$, hence $(S,D)=(Z,D)^r$.

That is, the operation $\underset{\theta}{\overset{*}{\sim}}$ does not have idempotency property on the set $S_E(U)$.

6) $(Z,D)\underset{\theta}{\overset{*}{\sim}}\emptyset_D=\emptyset_D\underset{\theta}{\overset{*}{\sim}}(Z,D)=(Z,D)^r$.

Proof: Let $\emptyset_D=(S,D)$. Hence, $\forall t\in D$; $S(t)=\emptyset$. Let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,D)=(Y,D)$. Then, $\forall t\in D$,

$$Y(t)=\begin{cases} Z'(t), & t\in D\setminus D=\emptyset \\ Z'(t)\cap S'(t), & t\in D\cap D=D \end{cases}$$

Thus, $\forall t\in D$, $S(t)=Z'(t)\cap S'(t)=Z'(t)\cap U=Z'(t)$. Hence $(Y,D)=(P,D)^r$.

7) $(Z,D)\underset{\theta}{\overset{*}{\sim}}\emptyset_F=(Z,D)^r$.

Proof: Let $\emptyset_F=(S,F)$. Hence, $\forall t\in F$, $S(t)=\emptyset$. Let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)=(Y,D)$. Thus, $\forall t\in D$

$$Y(t)=\begin{cases} Z'(t), & t\in D\setminus F \\ Z'(t)\cap S'(t), & t\in D\cap F \end{cases}$$

Thus, $\forall t\in D$, $Y(t)=Z'(t)=Z'(t)\cap S'(t)=Z'(t)\cap U=Z'(t)$ and thus $(Y,D)=(Z,D)^r$.

8) $(Z,D)\underset{\theta}{\overset{*}{\sim}}\emptyset_E=(Z,D)^r$.

Proof: Let $\emptyset_E$ =(S,E). Hence $\forall t\in E$; S(t)=$\emptyset$. Let (Z,D) $\underset{\theta}{\overset{*}{\sim}}$ (S,E)=(Y,D). Thus, $\forall t\in D$,

$$Y(t)=\begin{cases} Z'(t), & t\in D\backslash E=\emptyset \\ Z'(t)\cap S'(t), & t\in D\cap E=D \end{cases}$$

Hence, $\forall t\in D$ S(t)=Z'(t)∩S'(t)=Z'(t)∩U=Z'(t), so (Y,D)=$(Z,D)^r$.

9) (Z,D) $\underset{\theta}{\overset{*}{\sim}}$ U_D=U_D $\underset{\theta}{\overset{*}{\sim}}$ (Z,D)=$\emptyset_D$.

Proof: Let U_D=(T,D). Hence, $\forall t\in D$, T(t)=U. Let (Z,D) $\underset{\theta}{\overset{*}{\sim}}$ (T, D)=(S,D). Hence, $\forall t\in D$;

$$S(t)=\begin{cases} Z'(t), & t\in D\backslash D=\emptyset \\ Z'(t)\cap T'(t) & t\in D\cap D=D \end{cases}$$

Hence, $\forall t\in D$ S(t)=Z'(t)∩T'(t)=Z'(t)∩$\emptyset$=$\emptyset$, so (S,D)=$\emptyset_D$.

10) (Z,D) $\underset{\theta}{\overset{*}{\sim}}$ U_F=$(Z,D\backslash F)^r$.

Proof: Let U_F=(T,F). Hence, $\forall t\in F$, T(t)=U. Let (Z,D) $\underset{\theta}{\overset{*}{\sim}}$ (T, F)=(S,D). So, $\forall t\in D$,

$$S(t)=\begin{cases} Z'(t), & t\in D\backslash F \\ Z'(t)\cap T'(t), & t\in D\cap F \end{cases}$$

Hence,

$$S(t)=\begin{cases} Z'(t), & t\in D\backslash F \\ Z'(t)\cap\emptyset, & t\in D\cap F \end{cases}$$

Thus, $\forall t\in D\backslash F$, S(t)=Z'(t), so (S,D)=$(Z,D\backslash F)^r$.

11) U_F $\underset{\theta}{\overset{*}{\sim}}$ (Z,D)=$\emptyset_F$.

Proof: Let U_F=(T,F). Hence $\forall t\in F$, T(t)=U. Let (T,F) $\underset{\theta}{\overset{*}{\sim}}$ (Z,D)=(S,F), so $\forall t\in F$,

$$S(t)=\begin{cases} T'(t), & t\in F\setminus D \\ T'(t)\cap Z'(t), & t\in F\cap D \end{cases}$$

Hence,

$$S(t)=\begin{cases} \emptyset, & t\in F\setminus D \\ \emptyset, & t\in F\cap D \end{cases}$$

Thus, $\forall t\in F$, $S(t)=\emptyset$, so $(S,F)=\emptyset_F$.

12) $(Z,D)\underset{\theta}{\overset{*}{\sim}} U_E=\emptyset_D$.

Proof: Let $U_E=(T,E)$. Hence, $\forall t\in E$, $T(t)=U$. Let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(T,E)=(S,D)$, then $\forall t\in D$;

$$S(t)=\begin{cases} Z'(t), & t\in D\setminus E=\emptyset \\ Z'(t)\cap T'(t), & t\in D\cap E=D \end{cases}$$

Hence, $\forall t\in D$, $S(t)=Z'(t)\cap T'(t)=Z'(t)\cap\emptyset=\emptyset$, so $(S,D)=\emptyset_D$

13) $U_E\underset{\theta}{\overset{*}{\sim}}(Z,D)=\emptyset_E$.

Proof: Let $U_E=(T,E)$. Thus, $\forall t\in E$, $T(t)=U$. Let $(T,E)\underset{\theta}{\overset{*}{\sim}}(Z,D)=(S,E)$, so $\forall t\in E$,

$$S(t)=\begin{cases} T'(t), & t\in E\setminus D \\ T'(t)\cap Z'(t), & t\in E\cap D \end{cases}$$

Hence,

$$S(t)=\begin{cases} \emptyset, & t\in E\setminus D \\ \emptyset, & t\in E\cap D \end{cases}$$

Thus, $\forall t\in E$, $S(t)=\emptyset$, so $(S,E)=\emptyset_E$.

14) $(Z,D)\underset{\theta}{\overset{*}{\sim}}(Z,D)^r=(Z,D)^r\underset{\theta}{\overset{*}{\sim}}(Z,D)=\emptyset_D$.

Proof: Let $(Z,D)^r=(S,D)$, so $\forall t\in D$, $S(t)=Z'(t)$. Let $(D,A)\underset{\theta}{\overset{*}{\sim}}(S,D)=(T,D)$, so $\forall t\in D$,

$$T(t)=\begin{cases} Z'(t), & t\in D\setminus D=\emptyset \\ Z'(t)\cap S'(t), & t\in D\cap D=D \end{cases}$$

Hence, $\forall t\in D$, $T(t)=Z'(t)\cap S'(t)=Z'(t)\cap Z(t)=\emptyset$, so $(T,D)=\emptyset_D$

15) $[(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)]^r=(Z,D)\widetilde{\cup}(R,J)$.

Proof: Let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)=(S,D)$. Then, $\forall t\in D$,

$$S(t)=\begin{cases} Z'(t), & t\in D\setminus J \\ Z'(t)\cap R'(t), & t\in D\cap J \end{cases}$$

Let $(S,D)^r=(T,D)$, so $\forall t\in D$,

$$T(t)=\begin{cases} Z(t), & t\in D\setminus J \\ Z(t)\cup R(t), & t\in D\cap J \end{cases}$$

Thus, $(T,D)=(Z,D)\widetilde{\cup}(R,J)$.

In classical theory, $A\cap B=U \Leftrightarrow A=U$ and $B=U$. Now, we have the following:

16) $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)=U_D \Leftrightarrow (Z,D)=\emptyset_D$ and $(R,J)=\emptyset_{D\cap J}$.

Proof: Let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)=(T,D)$. Hence, $\forall t\in D$,

$$T(t)=\begin{cases} Z'(t), & t\in D\setminus J \\ Z'(t)\cap R'(t), & t\in D\cap J \end{cases}$$

Since $(T,D)=U_D$, $\forall t\in D$, $T(t)=U$. Hence, $\forall t\in D\setminus J$, $Z'(t)=U$, thus $Z(t)=\emptyset$ and $\forall t\in D\cap J$, $T(t)=Z'(t)\cap R'(t)=U \Leftrightarrow \forall t\in D\cap J$, $Z'(t)=U$ and $R'(t)=U$ $\Leftrightarrow \forall t\in D$, $Z(t)=\emptyset$ and for $\forall t\in D\cap E$, $R(t)=\emptyset \Leftrightarrow (Z,D)=\emptyset_D$ and $(R,J)=\emptyset_{D\cap J}$.

17) $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,D)=U_D \Leftrightarrow (Z,D)=(R,D)=\emptyset_D$.

Proof: Let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,D)=(T,D)$. Hence, $\forall t\in D$,

$$T(t)=\begin{cases} Z'(t), & t\in D\setminus D=\emptyset \\ Z'(t)\cap R'(t), & t\in D\cap D=D \end{cases}$$

Since $(T,D)=U_D$, $\forall t\in D$, $T(t)=U$. Hence, $\forall t\in D$, $T(t)=Z'(t)\cap R'(t)=U \Leftrightarrow \forall t\in D$, $Z'(t)=U$ and $R'(t)=U \Leftrightarrow \forall t\in D$, $Z(t)=\emptyset$ and $R(t)=\emptyset \Leftrightarrow (Z,D)=(R,D)=\emptyset_D$.

In classical theory, for all A, $\emptyset\subseteq A$. Now, we have the following:

18) $\emptyset_D \tilde{\subseteq} (Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)$ and $\emptyset_J \tilde{\subseteq} (R,J)\underset{\theta}{\overset{*}{\sim}}(Z,D)$.

In classical theory, for all A, $A\subseteq U$. Now, we have the following:

19) $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)\tilde{\subseteq}\ U_D$ and $(R,J)\underset{\theta}{\overset{*}{\sim}}(Z,D)\tilde{\subseteq}\ U_J$

In classical theory, for all $D\cap J\subseteq D$ (and all $D\cap J\subseteq J$). Now, we have the following:

20) $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)\tilde{\subseteq}(Z,D)^r$, however $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)$ needs not to be a soft subset of $(R,J)^r$.

Proof: Let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)=(S,D)$. First of all, $D\subseteq D$. Moreover, $\forall t\in D$,

$$S(t)=\begin{cases} Z'(t), & t\in D\setminus J \\ Z'(t)\cap R'(t), & t\in D\cap J \end{cases}$$

Since $\forall t\in D\setminus J$, $Z'(t)\subseteq Z'(t)$ and $\forall t\in D\cap J$, $Z'(t)\cap R'(t)\subseteq Z'(t)$, hence $\forall t\in D$, $S(t)\subseteq Z'(t)$.

Therefore, $(S,D)=(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)\tilde{\subseteq}(Z,D)^r$.

21) $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,D)\tilde{\subseteq}(Z,D)^r$, moreover $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,D)\tilde{\subseteq}(R,D)^r$.

Proof: Let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,D)=(S,D)$. First of all, $D\subseteq D$. Moreover, $\forall t\in D$,

$$S(t)=\begin{cases} Z'(t), & t\in D\setminus D=\emptyset \\ Z'(t)\cap R'(t), & t\in D\cap D=D \end{cases}$$

Since $\forall t\in D$, $S(t)=Z'(t)\cap R'(t)\subseteq Z'(t)$, so $(S,D)=(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,D)\tilde{\subseteq}(Z,D)^r$. $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,D)\tilde{\subseteq}(R,D)^r$ can be shown similarly.

4. Distribution Rules

In this section, distribution of complementary soft binary piecewise theta (θ) operation over other soft set operations such as extended soft set operations, complementary extended soft set operations, restricted soft set operations, soft binary piecewise operations and complementary soft binary piecewise operation are examined in detail and many interesting results are obtained.

4.1. Distribution of complementary soft binary piecewise theta (θ) operation over extended soft set operations:

i) Left-distribution of complementary soft binary piecewise theta (θ) operation over extended soft set operations:

1) $(Z,D)\underset{\theta}{\overset{*}{\sim}}[(R,J)\cap_{\varepsilon}(S,F)]=[(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)]\cap_{\varepsilon}[(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)]$, where $D\cap J\cap F=\emptyset$.

Proof: Let $(R,J)\cap_{\varepsilon}(S,F)=(M,J\cup F)$, so $\forall t\in J\cup F$,

$$M(t)=\begin{cases} R(t), & t\in J\setminus F \\ S(t), & t\in F\setminus J \\ R(t)\cap S(t), & t\in J\cap F \end{cases}$$

Let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(M,J\cup F)=(N,D)$, $\forall t\in D$,

$$N(t)=\begin{cases} Z'(t), & t\in D\setminus(J\cup F) \\ Z'(t)\cap M'(t), & t\in D\cap(J\cup F) \end{cases}$$

Thus,

$$N(t)=\begin{cases} Z'(t), & t\in D\setminus(J\cup F)=D\cap J'\cap F' \\ Z'(t)\cap R'(t), & t\in D\cap(J\setminus F)=D\cap J\cap F' \\ Z'(t)\cap S'(t), & t\in D\cap(F\setminus J)=D\cap J'\cap F \\ Z'(t)\cap[(R'(t)\cup S'(t)], & t\in D\cap J\cap F=D\cap J\cap F \end{cases}$$

Now let's handle the right hand side of the equality. Assume that $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)=(V,D)$, then for $\forall t\in D$,

$$V(t)=\begin{cases} Z'(t), & t\in D\setminus J \\ Z'(t)\cap R'(t), & t\in D\cap J \end{cases}$$

Now let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)=(W,D)$. Then, $\forall t\in D$,

$$W(t)=\begin{cases} Z'(t), & t\in D\backslash F \\ Z'(t)\cap S'(t), & t\in D\cap F \end{cases}$$

Assume that $(V,D)\cap_{\varepsilon}(W,D)=(T,D)$, then $\forall t\in D$,

$$T(t)=\begin{cases} V(t), & t\in D\backslash D=\emptyset \\ W(t), & t\in D\backslash D=\emptyset \\ V(t)\cap W(t), & t\in D\cap D=D \end{cases}$$

Hence,

$$T(t)=\begin{cases} Z'(t)\cap Z'(t), & t\in(D\backslash J)\cap(D\backslash F) \\ Z'(t)\cap[Z'(t)\cap S'(t)], & t\in(D\backslash J)\cap(D\cap F) \\ [Z'(t)\cap R'(t)]\cap Z'(t), & t\in(D\cap J)\cap(D\backslash F) \\ [Z'(t)\cap R'(t)]\cap[Z'(t)\cap S'(t)], & t\in(D\cap J)\cap(D\cap F) \end{cases}$$

Thus,

$$T(t)=\begin{cases} Z'(t), & t\in D\cap J'\cap F' \\ Z'(t)\cap S'(t), & t\in D\cap J'\cap F \\ Z'(t)\cap R'(t), & t\in D\cap J\cap F' \\ [Z'(t)\cap R'(t)]\cap[Z'(t)\cap S'(t)], & t\in D\cap J\cap F \end{cases}$$

It is seen that N=T.

2) $(Z,D)\underset{\theta}{\overset{*}{\sim}}[(R,J)\cup_{\varepsilon}(S,F)]=[(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)]\cap_{\varepsilon}[(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)]$, where $D\cap J\cap F=\emptyset$.

3) $(Z,D)\underset{\theta}{\overset{*}{\sim}}[(R,J)\lambda_{\varepsilon}(S,F)]=[(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)]\tilde{\cap}[(S,F)\underset{\backslash}{\overset{*}{\sim}}(Z,D)]$, where $D\cap J'\cap F=\emptyset$.

4) $(Z,D)\underset{\theta}{\overset{*}{\sim}}[(R,J)\backslash_{\varepsilon}(S,F)]=[(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)]\tilde{\cup}[(S,F)\underset{\backslash}{\overset{*}{\sim}}(Z,D)]$, where $D\cap J'\cap F=\emptyset$.

ii) Right-distribution of complementary soft binary piecewise theta (θ) operation over extended soft set operations:

1) $[(Z,D)\cup_{\varepsilon}(R,J)]\underset{\theta}{\overset{*}{\sim}}(S,F)=[(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)]\cap_{\varepsilon}[(R,J)\underset{\theta}{\overset{*}{\sim}}(S,F)]$.

Proof: Let $(Z,D)\cap_{\varepsilon}(R,J)=(M,D\cup J)$, so $\forall t\in D\cup J$,

$$M(t)=\begin{cases} Z(t), & t\in D\setminus J \\ R(t), & t\in J\setminus D \\ Z(t)\cup R(t), & t\in D\cap J \end{cases}$$

Let $(M,D\cup J)\underset{\theta}{\overset{*}{\sim}}(S,F)=(N,D\cup J)$, so $\forall t\in D\cup J$,

$$N(t)=\begin{cases} M'(t), & t\in (D\cup J)\setminus F \\ M'(t)\cap S'(t), & t\in (D\cup J)\cap F \end{cases}$$

Thus,

$$N(t)=\begin{cases} Z'(t), & t\in (D\setminus J)\setminus F=D\cap J'\cap F' \\ R'(t), & t\in (J\setminus D)\setminus F=D'\cap J\cap F' \\ Z'(t)\cap R'(t), & t\in (D\cap J)\setminus F=D\cap J\cap F' \\ Z'(t)\cap S'(t), & t\in (D\setminus J)\cap F=D\cap J'\cap F \\ R'(t)\cap S'(t), & t\in (J\setminus D)\cap F=D'\cap J\cap F \\ [Z'(t)\cap R'(t)]\cap S'(t), & t\in (D\cap J)\cap F=D\cap J\cap F \end{cases}$$

Now let's handle the right hand side of the equality. Let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)=(V,D)$, so $\forall t\in D$,

$$V(t)=\begin{cases} Z'(t), & t\in D\setminus F \\ Z'(t)\cap S'(t), & t\in D\cap F \end{cases}$$

Let $(R,J)\underset{\theta}{\overset{*}{\sim}}(S,F)=(W,J)$, so $\forall t\in J$,

$$W(t)=\begin{cases} R'(t), & t\in J\setminus F \\ R'(t)\cap S'(t), & t\in J\cap F \end{cases}$$

Assume that $(V,D)\cap_{\varepsilon}(W,J)=(T,D\cup J)$, so $\forall t\in D\cup J$,

$$T(t)=\begin{cases} V(t), & t\in D\setminus J \\ W(t), & t\in J\setminus D \\ V(t)\cap W(t), & t\in D\cap J \end{cases}$$

Thus,

$$T(t)=\begin{cases} Z'(t), & t\in(D\backslash F)\backslash J=D\cap J'\cap F' \\ Z'(t)\cap S'(t), & t\in(D\cap F)\backslash J=D\cap J'\cap F \\ R'(t), & t\in(J\backslash F)\backslash D=D'\cap J\cap F' \\ R'(t)\cap S'(t), & t\in(J\cap F)\backslash D=D'\cap J\cap F \\ Z'(t)\cap R'(t), & t\in(D\backslash F)\cap(J\backslash F)=D\cap J\cap F' \\ Z'(t)\cap[R'(t)\cap S'(t)], & t\in(D\backslash F)\cap(J\cap F)=\emptyset \\ [Z'(t)\cap S'(t)]\cap R'(t), & t\in(D\cap F)\cap(J\backslash F)=\emptyset \\ [Z'(t)\cap S'(t)]\cap [R'(t)\cap S'(t)], & t\in(D\cap F)\cap(J\cap F)=D\cap J\cap F \end{cases}$$

It is seen that N=T.

2) $[(Z,D)\cap_\varepsilon (R,J)]\overset{*}{\underset{\theta}{\sim}}(S,F)=[(Z,D)\overset{*}{\underset{\theta}{\sim}}(S,F)]\cup_\varepsilon[(R,J)\overset{*}{\underset{\theta}{\sim}}(S,F)]$

3) $[(Z,D)\backslash_\varepsilon (R,J)]\overset{*}{\underset{\theta}{\sim}}(S,F)=[(Z,D)\overset{*}{\underset{\theta}{\sim}}(S,F)]\cup_\varepsilon[(R,J)\overset{*}{\underset{\backslash}{\sim}}(S,F)]$,where $D\cap J\cap F'=D'\cap J\cap F=\emptyset$.

4) $[(Z,D)\lambda_\varepsilon(R,J)]\overset{*}{\underset{\theta}{\sim}}(S,F)=[(Z,D)\overset{*}{\underset{\theta}{\sim}}(S,F)]\cap_\varepsilon[(R,J)\overset{*}{\underset{\backslash}{\sim}}(S,F)]$, where $D\cap J\cap F'=D'\cap J\cap F=\emptyset$.

4.2. Distribution of complementary soft binary piecewise theta (θ) operation over complementary extended soft set operations:

i) Left-distribution of complementary soft binary piecewise theta (θ) operation over complementary extended soft set operations:

1) $(Z,D)\overset{*}{\underset{\theta}{\sim}}[(R,J)\overset{*}{\theta}_\varepsilon(S,F)]=[(Z,D)\overset{*}{\underset{\gamma}{\sim}}(R,J)]\cap_\varepsilon[(Z,D)\overset{*}{\underset{\gamma}{\sim}}(S,F)]$ where $D\cap J\cap F=\theta$.

Proof: Assume $(R,J)\overset{*}{\theta}_\varepsilon(S,F)=(M,J\cup F)$, so $\forall t\in J\cup F$,

$$M(t)=\begin{cases} R'(t), & t\in J\backslash F \\ S'(t), & t\in F\backslash J \\ R'(t)\cap S'(t), & t\in J\cap F \end{cases}$$

Let $(Z,D)\overset{*}{\underset{\theta}{\sim}}(M,J\cup F)=(N,D)$, then $\forall t\in D$,

$$N(t)=\begin{cases} Z'(t), & t\in D\backslash(J\cup F) \\ Z'(t)\cap M'(t), & t\in D\cap(J\cup F) \end{cases}$$

Hence,

$$N(t)=\begin{cases} Z'(t), & t\in D\setminus (J\cup F)=D\cap J'\cap F' \\ Z'(t)\cap R(t), & t\in D\cap (J\setminus F)=D\cap J\cap F' \\ Z'(t)\cap S(t), & t\in D\cap (C\setminus J)=D\cap J'\cap F \\ Z'(t)\cap [(R(t)\cup S(t)], & t\in D\cap J\cap F=D\cap J\cap F \end{cases}$$

Now let's handle the right hand side of the equality, $[(Z,D)\overset{*}{\underset{\gamma}{\sim}}(R,J)]\cap_{\varepsilon}[(Z,D)\overset{*}{\underset{\gamma}{\sim}}(S,F)]$. Let

$(Z,D)\overset{*}{\underset{\gamma}{\sim}}(R,J)=(V,D)$, so $\forall t\in D$,

$$V(t)=\begin{cases} Z'(t), & t\in D\setminus J \\ Z'(t)\cap R(t), & t\in D\cap J \end{cases}$$

Let $(Z,D)\overset{*}{\underset{\gamma}{\sim}}(S,F)=(W,D)$, hence $\forall t\in D$,

$$W(t)=\begin{cases} Z'(t), & t\in D\setminus F \\ Z'(t)\cap S(t), & t\in D\cap F \end{cases}$$

Assume that $(V,D)\cap_{\varepsilon}(W,D)=(T,D)$, hence $\forall t\in D$,

$$T(t)=\begin{cases} V(t), & t\in D\setminus D=\emptyset \\ W(t), & t\in D\setminus D=\emptyset \\ V(t)\cap W(t), & t\in D\cap D=D \end{cases}$$

Hence,

$$T(t)=\begin{cases} Z'(t)\cap Z'(t), & t\in (D\setminus J)\cap (D\setminus F) \\ Z'(t)\cap [Z'(t)\cap S(t)], & t\in (D\setminus J)\cap (D\cap J) \\ [Z'(t)\cap R(t)]\cap Z'(t), & t\in (D\cap J)\cap (D\setminus F) \\ [Z'(t)\cap R(t)]\cap [Z'(t)\cap S(t)], & t\in (D\cap J)\cap (D\cap F) \end{cases}$$

Thus,

$$T(t)=\begin{cases} Z'(t), & t\in D\cap J'\cap F' \\ Z'(t)\cap S(t), & t\in D\cap J'\cap F \\ Z'(t)\cap R(t), & t\in D\cap J\cap F' \\ [Z'(t)\cap R(t)]\cap [Z'(t)\cap S(t)], & t\in D\cap J\cap F \end{cases}$$

It is seen that N=T.

2) $(Z, D) \underset{\theta}{\overset{*}{\sim}} [(R,J) \overset{*}{*}_{\varepsilon} (S,F)]=[(Z,D) \underset{\gamma}{\overset{*}{\sim}} (R,J)]\cap_{\varepsilon}[(Z,D) \underset{\gamma}{\overset{*}{\sim}} (S,F)]$ where $D\cap J\cap F=\emptyset$.

3) $(Z,D) \underset{\theta}{\overset{*}{\sim}} [(R,J) \overset{*}{+}_{\varepsilon} (S,F)]=[(Z,D) \underset{\gamma}{\overset{*}{\sim}} (R,J)]\tilde{\cap}[(S,F) \underset{\theta}{\overset{*}{\sim}} (Z,D)]$ where $D\cap J'\cap F=\emptyset$.

4) $(Z,D) \underset{\theta}{\overset{*}{\sim}} [(R,J) \overset{*}{\gamma}_{\varepsilon} (S,F)]=[(Z,D) \underset{\gamma}{\overset{*}{\sim}} (R,J)]\tilde{\cup}[(S,F) \underset{\theta}{\overset{*}{\sim}} (Z,D)]$ where $D\cap J'\cap F=\emptyset$.

ii)Right-distribution of complementary soft binary piecewise theta (θ) operation over complementary extended soft set operations:

1) $[(Z,D) \overset{*}{*}_{\varepsilon} (R,J)] \underset{\theta}{\overset{*}{\sim}} (S,F)=[(Z,D)\tilde{\setminus}(S,F)]\cap_{\varepsilon}[(R,J)\tilde{\setminus}(S,F)]$

Proof: Let first handle the left hand side of the equality, assume $(Z,D) \overset{*}{*}_{\varepsilon} (R,J)=(M,D\cup J)$ and $\forall t\in D\cup J$,

$$M(t)=\begin{cases} Z'(t), & t\in D\setminus J \\ R'(t), & t\in J\setminus D \\ Z'(t)\cup R'(t), & t\in D\cap J \end{cases}$$

Let $(M,D\cup J) \underset{\theta}{\overset{*}{\sim}} (S,F)=(N,D\cup J)$and $\forall t\in D\cup J$,

$$N(t)=\begin{cases} M'(t), & t\in (D\cup J)\setminus F \\ M'(t)\cap S'(t), & t\in (D\cup J)\cap F \end{cases}$$

Thus,

$$N(t)=\begin{cases} Z(t), & t\in (D\setminus J)\setminus F=D\cap J'\cap F' \\ R(t), & t\in (J\setminus D)\setminus F=D'\cap J\cap F' \\ Z(t)\cap R(t), & t\in (D\cap J)\setminus F=D\cap J\cap F' \\ Z(t)\cap S'(t), & t\in (D\setminus J)\cap F=D\cap J'\cap F \\ R(t)\cap S'(t), & t\in (J\setminus D)\cap F=D'\cap J\cap F \\ [Z(t)\cap R(t)]\cap S'(t), & t\in (D\cap J)\cap F=D\cap J\cap F \end{cases}$$

Now let's handle the right hand side of the equality, that is $[(Z,D)\tilde{\setminus}(S,F)]\cap_{\varepsilon}[(R,J)\tilde{\setminus}(S,F)]$. Let $(Z,D)\tilde{\setminus} (S,F)=(V,D)$and $\forall t\in D$,

$$V(t)=\begin{cases} Z(t), & t\in D\setminus F \\ Z(t)\cap S'(t), & t\in D\cap F \end{cases}$$

Let $(R,J)\tilde{\setminus}(S,F)=(W,J)$ and $\forall t\in J$,

$$W(t)=\begin{cases} R(t), & t\in J\setminus F \\ R(t)\cap S'(t), & t\in J\cap F \end{cases}$$

Assume that $(V,D)\cap_{\varepsilon}W,J)=(T,D\cup J)$ and $\forall t\in D\cup J$,

$$T(t)=\begin{cases} V(t), & t\in D\setminus J \\ W(t), & t\in J\setminus D \\ V(t)\cap W(t), & t\in D\cap J \end{cases}$$

Hence,

$$T(t)=\begin{cases} Z(t), & t\in(D\setminus F)\setminus J=D\cap J'\cap F' \\ Z(t)\cap S'(t) & t\in(D\cap F)\setminus J=D\cap J'\cap F \\ R(t), & t\in(J\setminus F)\setminus D=D'\cap J\cap F' \\ R(t)\cap S'(t), & t\in(J\cap F)\setminus D=D'\cap J\cap F \\ Z(t)\cap R(t), & t\in(D\setminus F)\cap(J\setminus F)=D\cap J\cap F' \\ Z(t)\cap[R(t)\cap S'(t)], & t\in(D\setminus F)\cap(J\cap F)=\emptyset \\ [Z(t)\cap S'(t)]\cap R(t), & t\in(D\cap F)\cap(J\setminus F)=\emptyset \\ [Z(t)\cap S'(t)]\cap[R(t)\cap S'(t)], & t\in(D\cap F)\cap(J\cap F)=D\cap J\cap F \end{cases}$$

It is seen that N=T.

2) $[(Z,D)\overset{*}{\theta_{\varepsilon}}(R,J)]\overset{*}{\underset{\theta}{\sim}}(S,F)=[(Z,D)\tilde{\setminus}(S,F)]\cup_{\varepsilon}[(R,J)\tilde{\setminus}(S,F)]$

3) $[(Z,D)\overset{*}{\gamma_{\varepsilon}}(R,J)]\overset{*}{\underset{\theta}{\sim}}(S,F)=[(Z,D)\tilde{\setminus}(S,F)]\cup_{\varepsilon}[(R,J)\tilde{\ }_{\theta}(S,F)]$ where $D\cap J\cap F'=D'\cap J\cap F=\emptyset$.

4) $[(Z,D)\overset{*}{+_{\varepsilon}}(R,J)]\overset{*}{\underset{\theta}{\sim}}(S,F)=[(Z,D)\tilde{\setminus}(S,F)]\cap_{\varepsilon}[(R,J)\tilde{\setminus}(S,F)]$, where $D\cap J\cap F'=D\cap J\cap F=\emptyset$.

4.3. Distribution of complementary soft binary piecewise theta (θ)operation over soft binary piecewise operations:

i)Left-distribution of complementary soft binary piecewise theta (θ)operation over soft binary piecewise operations:

1) $(Z,D)\overset{*}{\underset{\theta}{\sim}}[(R,J)\tilde{\cup}(S,F)]=[(Z,D)\overset{*}{\underset{\theta}{\sim}}(R,J)]\tilde{\cap}[(S,F)\overset{*}{\underset{\theta}{\sim}}(Z,D)]$.

Proof: Let $(R,J)\tilde{\cup}(S,F)=(M,J)$, so $\forall t\in J$,

$$M(t)=\begin{cases} R(t), & t\in J\backslash F \\ R(t)\cup S(t), & t\in J\cap F \end{cases}$$

$(Z,D)\underset{\theta}{\overset{*}{\sim}}(M,J)=(N,D)$, where $\forall t\in D$;

$$N(t)=\begin{cases} Z'(t), & t\in D\backslash J \\ Z'(t)\cap M'(t), & t\in D\cap J \end{cases}$$

Thus,

$$N(t)=\begin{cases} Z'(t), & t\in D\backslash J \\ Z'(t)\cap R'(t), & t\in D\cap(J\backslash F)=D\cap J\cap F' \\ Z'(t)\cap[R'(t)\cap S'(t)], & t\in D\cap J\cap F=D\cap J\cap F \end{cases}$$

Now let's handle the right hand side of the equality: $[(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)]\tilde{\cap}[(S,F)\underset{\theta}{\overset{*}{\sim}}(Z,D)]$. Assume that $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)=(V,D)$, then for $\forall t\in D$,

$$V(t)=\begin{cases} Z'(t), & t\in D\backslash J \\ Z'(t)\cap R'(t), & t\in D\cap J \end{cases}$$

Now let $(S,F)\underset{\theta}{\overset{*}{\sim}}(Z,D)=(W,F)$. Then, $\forall t\in F$,

$$W(t)=\begin{cases} S'(t), & t\in F\backslash D \\ S'(t)\cap Z'(t), & t\in F\cap D \end{cases}$$

Assume that $(V,D)\tilde{\cap}(W,F)=(T,D)$, then $\forall t\in D$,

$$T(t)=\begin{cases} V(t), & t\in D\backslash F \\ V(t)\cap W(t), & t\in D\cap F \end{cases}$$

Thus,

$$
T(t)=\begin{cases}
Z'(t), & t\in(D\backslash J)\backslash F=D\cap J'\cap F' \\
Z'(t)\cap R'(t) & t\in(D\cap J)\backslash F=D\cap J'\cap F \\
Z'(t)\cap S'(t), & t\in(D\backslash J)\cap(F\backslash D)=\emptyset \\
Z'(t)\cap[S'(t)\cap Z'(t)], & t\in(D\backslash J)\cap(F\cap D)=D\cap J'\cap F \\
[Z'(t)\cap R'(t)]\cap S'(t) & t\in(D\cap J)\cap(F\backslash D)=\emptyset \\
[Z'(t)\cap R'(t)]\cap[S'(t)\cap Z'(t)], & t\in(D\cap J)\cap(J\cap D)=D\cap J\cap F
\end{cases}
$$

Thus,

$$
T(t)=\begin{cases}
Z'(t), & t\in(D\backslash J)\backslash F=D\cap J'\cap F' \\
Z'(t)\cap R'(t) & t\in(D\cap J)\backslash F=D\cap J\cap F' \\
Z'(t)\cap S'(t), & t\in(D\backslash J)\cap(F\backslash D)=\emptyset \\
Z'(t)\cap S'(t) & t\in(D\backslash J)\cap(F\cap D)=D\cap J'\cap F \\
[Z'(t)\cap R'(t)]\cap S'(t) & t\in(D\cap J)\cap(F\backslash D)=\emptyset \\
[Z'(t)\cap R'(t)]\cap[S'(t)\cap Z'(t)], & t\in(D\cap J)\cap(F\cap D)=D\cap J\cap F
\end{cases}
$$

It is seen that N=T.

2) $(Z,D)\underset{\theta}{\overset{*}{\sim}}[(R,J)\tilde{\cap}(S,F)]=[(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)]\tilde{\cup}[(S,F)\underset{\theta}{\overset{*}{\sim}}(Z,D)]$.

3) $(Z,D)\underset{\theta}{\overset{*}{\sim}}[(R,J)\tilde{\backslash}(S,F)]=[(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)]\tilde{\cup}[(S,F)\underset{\backslash}{\overset{*}{\sim}}(Z,D)]$.

4) $(Z,D)\underset{\theta}{\overset{*}{\sim}}[(R,J)\tilde{\lambda}(S,F)]=[(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)]\tilde{\cap}[(S,F)\underset{\backslash}{\overset{*}{\sim}}(Z,D)]$.

ii)Right-distribution o f complementary soft binary piecewise theta (θ)operation over soft binary piecewise operations:

1)$[(Z,D)\tilde{\cap}(R,J)]\underset{\theta}{\overset{*}{\sim}}(S,F)=[(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)]\tilde{\cup}[(R,J)\underset{\theta}{\overset{*}{\sim}}(S,F)]$

Proof: Suppose $(Z,D)\tilde{\cap}(R,J)=(M,D)$, so $\forall t\in D$ için,

$$
M(t)=\begin{cases}
Z(t), & t\in D\backslash J \\
Z(t)\cap R(t), & t\in D\cap J
\end{cases}
$$

Let $(M,D)\underset{\theta}{\overset{*}{\sim}}(S,F)=(N,D)$, so $\forall t\in D$,

$$N(t)=\begin{cases} M'(t), & t\in D\setminus F \\ M'(t)\cap S'(t), & t\in D\cap F \end{cases}$$

Thus,

$$N(t)=\begin{cases} Z'(t), & t\in(D\setminus J)\setminus F=D\cap J'\cap F' \\ Z'(t)\cup R'(t), & t\in(D\cap J)\setminus F=D\cap J\cap F' \\ Z'(t)\cap S'(t), & t\in(D\setminus J)\cap F=D\cap J'\cap F \\ [Z'(t)\cup R'(t)]\cap S'(t), & t\in(D\cap J)\cap F=D\cap J\cap F \end{cases}$$

Now let's handle the right hand side of the equality: $[(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)]\tilde{\cup}[(R,J)\underset{\theta}{\overset{*}{\sim}}(S,F)]$. Let

$(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)=(V,D)$, so $\forall t\in D$,

$$V(t)=\begin{cases} Z'(t), & t\in D\setminus F \\ Z'(t)\cap S'(t), & t\in D\cap F \end{cases}$$

Let $(R,J)\underset{\theta}{\overset{*}{\sim}}(S,F)=(W,J)$, so $\forall t\in J$,

$$W(t)=\begin{cases} R'(t), & t\in J\setminus F \\ R'(t)\cap S'(t), & t\in J\cap F \end{cases}$$

Assume that $(V,D)\tilde{\cup}(W,J)=(T,D)$, so $\forall t\in D$,

$$T(t)=\begin{cases} V(t), & t\in D\setminus J \\ V(t)\cup W(t), & t\in D\cap J \end{cases}$$

Hence,

$$T(t)=\begin{cases} Z'(t), & t\in(D\setminus F)\setminus J=D\cap J'\cap F' \\ Z'(t)\cap S'(t), & t\in(D\cap F)\setminus J=D\cap J'\cap F \\ Z'(t)\cup R'(t), & t\in(D\setminus F)\cap(J\setminus F)=D\cap J\cap F' \\ Z'(t)\cup[R'(t)\cap S'(t)], & t\in(D\setminus F)\cap(J\cap F)=\emptyset \\ [Z'(t)\cap S'(t)]\cup R'(t), & t\in(D\cap F)\cap(J\setminus F)=\emptyset \\ [Z'(t)\cap S'(t)]\cup[R'(t)\cap S'(t)], & t\in(D\cap F)\cap(J\cap F)=D\cap J\cap F \end{cases}$$

It is seen that N=T.

2) $[(Z,D)\tilde{\cup}(R,J)]\underset{\theta}{\overset{*}{\sim}}(S,F)=[(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)]\tilde{\cap}[(R,J)\underset{\theta}{\overset{*}{\sim}}(S,F)]$

3) $[(Z,D)\tilde{\lambda}(R,J)]\underset{\theta}{\overset{*}{\sim}}(S,F)=[(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)]\tilde{\cap}[(R,J)\tilde{\setminus}(S,F)]$.

4)$[(Z,D)\tilde{\setminus}(R,J)]\underset{\theta}{\overset{*}{\sim}}(S,F)=[(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)]\tilde{\cup}[(R,J)\tilde{\setminus}(S,F)]$.

4.4. Distribution of complementary soft binary piecewise theta(θ)operation over complementary soft binary piecewise operations:

i)Left-distribution of complementarysoft binary piecewise theta(θ)operation over complementary soft binary piecewise operations:

1)$(Z,D)\underset{\theta}{\overset{*}{\sim}}[(R,J)\underset{*}{\overset{*}{\sim}}(S,F)]=[(Z,D)\underset{\gamma}{\overset{*}{\sim}}(R,J)]\tilde{\cup}[(Z,D)\underset{\gamma}{\overset{*}{\sim}}(S,F)]$ where $D\cap J\cap F'=\emptyset$ and $D\cap J\cap F=\emptyset$.

Proof: Let first handle the left hand side of the equality, suppose $(R,J)\underset{*}{\overset{*}{\sim}}(S,F)=(M,J)$, so $\forall t\in J$,

$$M(t)=\begin{cases} R'(t), & t\in J\setminus F \\ R'(t)\cup S'(t), & t\in J\cap F \end{cases}$$

Let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(M,J)=(N,D)$, so $\forall t\in D$,

$$N(t)=\begin{cases} Z'(t), & t\in D\setminus J \\ Z'(t)\cap M'(t), & t\in D\cap J \end{cases}$$

Thus,

$$N(t)=\begin{cases} Z'(t), & t\in D\setminus J \\ Z'(t)\cap R(t), & t\in D\cap(J\setminus F)=D\cap J\cap F' \\ Z'(t)\cap[(R(t)\cap S(t)], & t\in D\cap J\cap F=D\cap J\cap F \end{cases}$$

Now let's handle the right hand side of the equality: $[(Z,D)\underset{\gamma}{\overset{*}{\sim}}(R,J)]\tilde{\cup}[(Z,D)\underset{\gamma}{\overset{*}{\sim}}(S,F)]$. Let

$(Z,D)\underset{\gamma}{\overset{*}{\sim}}(R,J)=(V,D)$, so $\forall t\in D$,

$$V(t)=\begin{cases} Z'(t), & t\in D\backslash J \\ Z'(t)\cap R(t), & t\in D\cap J \end{cases}$$

Let $(Z,D)\overset{*}{\underset{\gamma}{\sim}}(S,F)=(W,D)$, so $\forall t\in D$,

$$W(t)=\begin{cases} Z'(t), & t\in D\backslash F \\ Z'(t)\cap S(t), & t\in D\cap F \end{cases}$$

Assume that $(V,D)\tilde{\cup}(W,D)=(T,D)$, so $\forall t\in D$,

$$T(t)=\begin{cases} V(t), & t\in D\backslash D=\emptyset \\ V(t)\cup W(t), & t\in D\cap D=D \end{cases}$$

Thus,

$$T(t)=\begin{cases} Z'(t) & t\in(D\backslash J)\cap(D\backslash F)=D\cap J'\cap F' \\ Z'(t)\cup[Z'(t)\cap S(t)], & t\in(D\backslash J)\cap(D\cap F)=D\cap J'\cap F \\ [Z'(t)\cap R(t)]\cup Z'(t), & t\in(D\cap J)\cap(D\backslash F)=D\cap J\cap F' \\ [Z'(t)\cap R(t)]\cup[Z'(t)\cap S(t)], & t\in(D\cap J)\cap(D\cap F)=D\cap J\cap F \end{cases}$$

Thus,

$$T(t)=\begin{cases} Z'(t) & t\in(D\backslash J)\cap(D\backslash F)=D\cap J'\cap F' \\ Z'(t), & t\in(D\backslash J)\cap(D\cap F)=D\cap J'\cap F \\ Z'(t), & t\in(D\cap J)\cap(D\backslash F)=D\cap J\cap F' \\ [Z'(t)\cap R(t)]\cup[Z'(t)\cap S(t)], & t\in(D\cap J)\cap(D\cap F)=D\cap J\cap F \end{cases}$$

Here let's handle $t\in D\backslash J$ in the first equation. Since $D\backslash J=D\cap J'$, if $t\in J'$, then $t\in F\backslash J$ or $t\in(J\cup F)'$. Hence, if $t\in D\backslash J$, $t\in D\cap J'\cap F'$ or $t\in D\cap J'\cap F$. Thus, it is seen that N=T.

2) $(Z,D)\overset{*}{\underset{\theta}{\sim}}[(R,J)\overset{*}{\underset{\theta}{\sim}}(S,F)]=[(Z,D)\overset{*}{\underset{\gamma}{\sim}}(R,J)]\tilde{\cup}[(Z,D)\overset{*}{\underset{\gamma}{\sim}}(S,F)]$ where $D\cap J\cap F'=\emptyset$.

3) $(Z,D)\overset{*}{\underset{\theta}{\sim}}[(R,J)\overset{*}{\underset{\gamma}{\sim}}(S,F)]=[(Z,D)\overset{*}{\underset{\gamma}{\sim}}(R,J)]\tilde{\cup}[(S,F)\overset{*}{\underset{\theta}{\sim}}(Z,D)]$.

4) $(Z,D)\overset{*}{\underset{\theta}{\sim}}[(R,J)\overset{*}{\underset{+}{\sim}}(S,F)]=[(Z,D)\overset{*}{\underset{\gamma}{\sim}}(R,J)]\tilde{\cup}[(S,F)\overset{*}{\underset{\theta}{\sim}}(Z,D)]$

ii)Right-distribution of complementary soft binary piecewise theta (θ)operation over complementary soft binary piecewise operations :

1)$[(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)]\underset{\theta}{\overset{*}{\sim}}(S,F)=[(Z,D)\tilde{\setminus}(S,F)]\tilde{\cup}[(R,J)\tilde{\setminus}(S,F)]$

Proof: Let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)=(M,D)$, so $\forall t\in D$,

$$M(t)=\begin{cases} Z'(t), & t\in D\setminus J \\ Z'(t)\cap R'(t), & t\in D\cap J \end{cases}$$

Let $(M,D)\underset{\theta}{\overset{*}{\sim}}(S,F)=(N,D)$, so $\forall t\in D$,

$$N(t)=\begin{cases} M'(t), & t\in D\setminus F \\ M'(t)\cap S'(t), & t\in D\cap F \end{cases}$$

Thus,

$$N(t)=\begin{cases} Z(t), & t\in (D\setminus J)\setminus F=D\cap J'\cap F' \\ Z(t)\cup R(t), & t\in (D\cap J)\setminus F=D\cap J\cap F' \\ Z(t)\cap S'(t), & t\in (D\setminus J)\cap F=D\cap J'\cap F \\ [Z(t)\cup R(t)]\cap S'(t), & t\in D\cap J\cap F=D\cap J\cap F \end{cases}$$

Now let's handle the right hand side of the equality:$[(Z,D)\tilde{\setminus}(S,F)]\tilde{\cup}[(R,J)\tilde{\setminus}(S,F)]$. Let $(Z,D)\tilde{\setminus}(S,F)=(V,D)$, so $\forall t\in D$,

$$V(t)=\begin{cases} Z(t), & t\in D\setminus F \\ Z(t)\cap S'(t), & t\in D\cap F \end{cases}$$

Let $(R,J)\tilde{\setminus}(S,F)=(W,J)$, so $\forall t\in J$,

$$W(t)=\begin{cases} R(t), & t\in J\setminus F \\ R(t)\cap S'(t), & t\in J\cap F \end{cases}$$

Let $(V,D)\tilde{\cup}(W,J)=(T,D)$, so $\forall t\in D$,

$$T(t)=\begin{cases} V(t), & t\in D\setminus J \\ V(t)\cap W(t), & t\in D\cap J \end{cases}$$

Thus,

$$
T(t)=\begin{cases}
Z(t), & t\in(D\setminus F)\setminus J=D\cap J'\cap F' \\
Z(t)\cap S'(t), & t\in(D\cap F)\setminus J=D\cap J'\cap F \\
Z(t)\cup R(t), & t\in(D\setminus F)\cap(J\setminus F)=D\cap J\cap F' \\
Z(t)\cup[R(t)\cap S'(t)], & t\in(D\setminus F)\cap(J\cap F)=\emptyset \\
[Z(t)\cap S'(t)]\cup R(t), & t\in(D\cap F)\cap(J\setminus F)=\emptyset \\
[Z(t)\cap S'(t)]\cup[R(t)\cap S'(t)], & t\in(D\cap F)\cap(J\cap F)=D\cap J\cap F
\end{cases}
$$

It is seen that N=T.

2) $(Z,D)\underset{*}{\overset{*}{\sim}}(R,J)]\underset{\theta}{\overset{*}{\sim}}(S,F)=[(Z,D)\tilde{\setminus}(S,F)]\tilde{\cap}[(R,J)\tilde{\setminus}(S,F)]$

3) $[(Z,D)\underset{+}{\overset{*}{\sim}}(R,J)]\underset{\theta}{\overset{*}{\sim}}(S,F)=[(Z,D)\tilde{\setminus}(S,F)]\tilde{\cap}[(R,J)\underset{\setminus}{\overset{*}{\sim}}(S,F)]$ where$D\cap J\cap F=\emptyset$

4) $[(Z,D)\underset{\gamma}{\overset{*}{\sim}}(R,J)]\underset{\theta}{\overset{*}{\sim}}(S,F)=[(Z,D)\tilde{\setminus}(S,F)]\tilde{\cup}[(R,J)\tilde{\theta}(S,F)]$ where$D\cap J\cap F'=\emptyset$

4.5. Distribution of complementary soft binary piecewise theta (θ) operation over restricted soft set operations:

i) Left-distribution of complementary soft binary piecewise theta (θ) operation over restricted soft set operations:

1) $(Z,D)\underset{\theta}{\overset{*}{\sim}}[(R,J)\cap_R(S,F)]=[(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)]\cup_R[(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)]$.

Proof: Let first handle the left hand side of the equality, suppose $(R,J)\cap_R(S,F)=(M,J\cap F)$and so $\forall t\in J\cap F$, $M(t)=R(t)\cap S(t)$. Let $(Z,D)\underset{\theta}{\overset{*}{\sim}}(M,J\cap F)=(N,D)$, so $\forall t\in D$,

$$
N(t)=\begin{cases}
Z'(t), & t\in D\setminus(J\cap F) \\
Z'(t)\cap M'(t), & t\in D\cap(J\cap F)
\end{cases}
$$

Hence,

$$
N(t)=\begin{cases}
Z'(t), & t\in D\setminus(J\cap F) \\
Z'(t)\cap[R'(t)\cup S'(t)], & t\in D\cap(J\cap F)
\end{cases}
$$

Now let's handle the right hand side of the equality: $[(Z,D)\overset{*}{\underset{\theta}{\sim}}(R,J)]\cup_R[(Z,D)\overset{*}{\underset{\theta}{\sim}}(S,F)]$. Let

$(Z,D)\overset{*}{\underset{\theta}{\sim}}(R,J)=(V,D)$, so $\forall t\in D$,

$$V(t)=\begin{cases} Z'(t), & t\in D\backslash J \\ Z'(t)\cap R'(t), & t\in D\cap J \end{cases}$$

Let $(Z,D)\overset{*}{\underset{\theta}{\sim}}(S,F)=(W,D)$, so $\forall t\in D$,

$$W(t)=\begin{cases} Z'(t), & t\in D\backslash F \\ Z'(t)\cap S'(t), & t\in D\cap F \end{cases}$$

Assume that $(V,D)\cup_R(W,D)=(T,D)$, and so $\forall t\in D$, $T(t)=V(t)\cup W(t)$,

$$T(t)=\begin{cases} Z'(t)\cup Z'(t), & t\in(D\backslash J)\cap(D\backslash F) \\ Z'(t)\cup[Z'(t)\cap S'(t)], & t\in(D\backslash J)\cap(D\cap F) \\ [Z'(t)\cap R'(t)]\cup Z'(t), & t\in(D\cap J)\cap(D\backslash F) \\ [Z'(t)\cap R'(t)]\cup[Z'(t)\cap S'(t)], & t\in(D\cap J)\cap(D\cap F) \end{cases}$$

Hence,

$$T(t)=\begin{cases} Z'(t), & t\in D\cap J'\cap F' \\ Z'(t), & t\in D\cap J'\cap F \\ Z'(t), & t\in D\cap J\cap F' \\ [Z'(t)\cap R'(t)]\cup[Z'(t)\cap S'(t)], & t\in D\cap J\cap F \end{cases}$$

Consi dering the parameter set of the first equation of the first row, that is, $D\backslash(J\cap F)$; since $D\backslash(J\cap F)=D\cap(J\cap F)'$, an element in $(J\cap F)'$ may be in $J\backslash F$, in $F\backslash J$ or $(J\cup F)$. Then, $D\backslash(J\cap F)$ is equivalent to the following 3 states: $D\cap(J\cap F')$, $D\cap(J'\cap F)$and $D\cap(J'\cap F')$. Hence, (1)=(2)

2) $(Z,D)\overset{*}{\underset{\theta}{\sim}}[(R,J)\cup_R(S,F)]=[(Z,D)\overset{*}{\underset{\theta}{\sim}}(R,J)]\cup_R[(Z,D)\overset{*}{\underset{\theta}{\sim}}(S,F)]$ where $D\cap J\cap F=\emptyset$

3) $(Z,D)\overset{*}{\underset{\theta}{\sim}}[(R,J)\theta_R(S,F)]=[(Z,D)\overset{*}{\underset{\gamma}{\sim}}(R,J)]\cup_R[(Z,D)\overset{*}{\underset{\gamma}{\sim}}(S,F)]$.

4) $(Z,D)\overset{*}{\underset{\theta}{\sim}}[(R,J)*_R(S,F)]=[(Z,D)\overset{*}{\underset{\theta}{\sim}}(R,J)]\cup_R[(Z,D)\overset{*}{\underset{\theta}{\sim}}(S,F)]$, where $D\cap J\cap F=\emptyset$.

5) $(Z,D)\underset{\theta}{\overset{*}{\sim}}[(R,J)\gamma_R(S,F)]=[(Z,D)\underset{\gamma}{\overset{*}{\sim}}(R,J)]\cup_R[(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)]$, where $D\cap J\cap F=\emptyset$.

6) $(Z,D)\underset{\theta}{\overset{*}{\sim}}[(R,J)\lambda_R(S,F)]=[(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)]\cup_R[(Z,D)\underset{\gamma}{\overset{*}{\sim}}(S,F)]$, where $D\cap J\cap F=\emptyset$.

7) $[(Z,D)\underset{\theta}{\overset{*}{\sim}}[(R,J)\backslash_R(S,F)]=[(Z,D)\underset{\theta}{\overset{*}{\sim}}(R,J)]\cup_R[(Z,D)\underset{\gamma}{\overset{*}{\sim}}(S,F)]$.

8) $(Z,D)\underset{\theta}{\overset{*}{\sim}}[(R,J)+_R(S,F)]=[(Z,D)\underset{\gamma}{\overset{*}{\sim}}(R,J)]\cup_R[(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)]$, where $D\cap J\cap F=\emptyset$.

ii)Right-distribution of complementary soft binary piecewise theta (θ) operation over restricted soft set operations:

1)$[(Z,D)\cup_R(R,J)]\underset{\theta}{\overset{*}{\sim}}(S,F)=[(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)]\cap_R[(R,J)\underset{\theta}{\overset{*}{\sim}}(S,F)]$.

Proof: Let first handle the left hand side of the equality, suppose $(Z,D)\cup_R(R,J)=(M,D\cap J)$ so, $\forall t\in D\cap J$, $M(t)=Z(t)\cup R(t)$. Let $(M,D\cap J)\underset{\theta}{\overset{*}{\sim}}(S,F)=(N,D\cap J)$, so $\forall t\in D\cap J$,

$$N(t)=\begin{cases} M'(t), & t\in(D\cap J)\backslash F \\ M'(t)\cap S'(t), & t\in(D\cap J)\cap F \end{cases}$$

Hence,

$$N(t)=\begin{cases} Z'(t)\cap R'(t), & t\in(D\cap J)\backslash F=D\cap J\cap F' \\ [Z'(t)\cap R'(t)]\cap S'(t), & t\in(D\cap J)\cap F \end{cases}$$

Now let's handle the right hand side of the equality, $[(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)]\cap_R[(R,J)\underset{\theta}{\overset{*}{\sim}}(S,F)]$. Let

$(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)=(V,D)$, so $\forall t\in D$,

$$V(t)=\begin{cases} Z'(t), & t\in D\backslash F \\ Z'(t)\cap S'(t), & t\in D\cap F \end{cases}$$

Let $(R,J)\underset{\theta}{\overset{*}{\sim}}(S,F)=(W,J)$, so $\forall t\in J$,

$$W(t)=\begin{cases} R'(t), & t\in J\setminus F \\ R'(t)\cap S'(t), & t\in J\cap F \end{cases}$$

Suppose that $(V,D)\cap_R (W,J)=(T,D\cap J)$, so $\forall t\in D\cap J$, $T(t)=V(t)\cap W(t)$,

$$T(t)=\begin{cases} Z'(t)\cap R'(t), & t\in(D\setminus F)\cap(J\setminus F)=D\cap J\cap' F \\ Z'(t)\cap[R'(t)\cap S'(t)], & t\in(D\setminus F)\cap(J\cap F)=\emptyset \\ [Z'(t)\cap S'(t)]\cap R'(t), & t\in(D\cap F)\cap(J\setminus F)=\emptyset \\ [Z'(t)\cap S'(t)]\cap[R'(t)\cap S'(t)], & t\in(D\cap F)\cap(J\cap F)=D\cap J\cap F \end{cases}$$

It is seen that N=T.

2) $[(Z,D)\cap_R (R,J)]\underset{\theta}{\overset{*}{\sim}}(S,F)=[(Z,D)\underset{\theta}{\overset{*}{\sim}}(S,F)]\cup_R[(R,J)\underset{\theta}{\overset{*}{\sim}}(S,F)]$.

3) $(Z,D)*_R (R,J)]\underset{\theta}{\overset{*}{\sim}}(S,F)=[(Z,D)\tilde{\setminus}(S,F)]\cap_R[(R,J)\tilde{\setminus}(S,F)]$.

4) $[(Z,D)\theta_R (R,J)]\underset{\theta}{\overset{*}{\sim}}(S,F)=[(Z,D)\tilde{\setminus}(S,F)]\cup_R[(R,J)\tilde{\setminus}(S,F)]$.

5. Conclusion

The concept of soft set operations is an essential concept similar to fundamental operations on numbers and basic operations on sets. Soft set operations are the operations that are applied on two or more soft sets to develop a relationship between them. The operations in soft set theory have proceed under two main headings up to now, as restricted soft set operations and extended soft set operations. In this paper, we contribute to the soft set literature by defining a new kind of soft set operation which we call complementary soft binary piecewise theta operation. The basic algebraic properties of the operations are investigated. Moreover by examining the distribution rules, we obtain the relationships between this new soft set operation and other types of soft set operations such as extended soft set operations, complementary extended soft set operations, soft binary piecewise operations, complementary soft binary piecewise operations and restricted soft set operations.This paper can be regarded as a theoretical study for soft sets and some future studies may continue by examining the distributions of other soft set operations over complementary soft binary piecewise theta operation and some new types of soft set operations can be defined in the following studies. Also, this research is to serve as a basis for many applications, especially decision making cryptography. Since soft set is a powerful mathematical tool for uncertain object detection, with this study, researchers may suggest some new encryption based on soft sets and also studies on the soft algebraic structures may be handled again as regards the algebraic properties by the operation defined in this paper.

Author's Contribution

The contribution of authors is equal.

Conflict of Interest

The authors have declared that there ais no conflict of interests.

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