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A New Type of Extended Soft Set Operation: Complementary Extended Lambda Operation

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ABSTRACT

Soft set theory was proposed by Molodtsov in 1999 to model some problems involving uncertainty and it has a wide range of theoretical and practical applications. Soft set operations constitute the vital building block of soft set theory. Since its introduction, several kinds of soft set operations have been proposed. In this study, in order to advance the soft theory, a new soft set operation known as the complementary extended lambda operation is described in this study, and all of its characteristics are thoroughly examined, and to obtain the relationship of the operation with other soft set operations, the distribution of this operation over other type soft set operations are examined.

Yeni Tip Genişletilmiş Esnek Küme İşlemi: Tümleyenli Genişletilmiş Lamda İşlemi

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ÖZ

Esnek küme teorisi, 1999 yılında Molodtsov tarafından belirsizlik içeren bazı problemleri modellemek amacıyla ortaya atılmış olup, geniş bir teorik ve pratik uygulama alanına sahiptir. Esnek küme işlemleri esnek küme teorisinin önemli yapı taşını oluşturur. Başlangıçtan bu yana çeşitli türlerde esnek küme işlemleri tanımlanmıştır. Teoriye katkı sağlamak amacıyla bu çalışmada tümleyenli genişletilmiş lamda işlemi olarak adlandırılan yeni bir esnek küme işlemi tanımlanmış, tüm özellikleri ayrıntılı olarak ele alınmış ve işlemin diğer esnek küme işlemleriyle ilişkisinin elde edilmesi için, bu işlemlerin diğer tip esnek ayarlama işlemlerine göre dağılımı incelenmiştir.

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1. Introduction

Probability theory, interval mathematics, statistics, intuitive fuzzy set theory, and fuzzy set theory are some of the most well-known and commonly used mathematical theories for modeling uncertainty. One of the most prominent theories among these theories is the fuzzy set theory. Since this theory contains some structural difficulties, there has been a need for different theories. A fuzzy set is defined through its membership function. Since it is difficult to create a membership function for each case, the nature of the membership function is highly individualized. Therefore, there has been a need for a set theory independent of the formation of the membership function. The Soft Set Theory proposed by Molodstov (1999) has eliminated the problems arising from the membership function. Molodstov has transferred soft set theory to many areas of mathematics. Operations research, game theory, probability, measurement theory, continuously differentiable functions, Riemann's integration, and Perron's integration are the areas where soft set theory has been successfully used.

Soft set operations constitute the basis of soft set theory, as studies on both soft algebraic structures and soft decision-making methods are based on soft set operations. In this regard, Maji et al. (2003) started the inspiring studies on soft set operations. A more widely accepted definition of soft subset than the one defined by Maji et al. (2003) was proposed by Pei and Miao (2005). When the studies of soft set operations such as Maji et al. (2003), Ali et al. (2009), Ali et al. (2011), Sezgin and Atagün (2011), Sezgin et al. (2019), Stojanovic (2021) are examined, it is seen that soft set operations proceed under two separate headings as restricted and extended operations. Eren and Çalışıcı (2019) proposed a new form of soft set operation for the literature and Sezgin and Çalışıcı (2024) improved the work of Eren and Çalışıcı (2019) and studied the properties of the soft binary piecewise difference operation comparing it with the difference operation in classical sets. Çağman (2021) and Sezgin et al. (2023c) studied new binary set operations, and these operations were transferred to soft sets by Aybek (2024). Besides, some new forms of soft set operations, different from the restricted and extended forms of operations were introduced by various authors (Sarıalioğlu, 2024; Akbulut, 2024; Sezgin and Aybek, 2023; Sezgin and Akbulut, 2023; Sezgin and Dagtoros, 2023; Sezgin and Demirci, 2023; Sezgin and Sarıalioğlu, 2024; Sezgin and Yavuz, 2023a; Sezgin et al., 2023a; Sezgin et al., 2023b; Sezgin and Atagün, 2023; Sezgin and Çağman, 2024), and soft set operations, one of the most fundamental elements of soft set theory, have been studied by researchers since the theory was introduced.

Moreover, different types of soft eqaulities were defined and some important equivalance relations were obtained with these different types of soft equalites as Jun and Yang, 2011; Liu et al., 2012; Feng and Li, 2013; Abbas et al., 2014; Abbas et al., 2017; Al-shami, 2019; Alshasi and El-Shafe, 2020; Ali et al., 2022. Studying the soft algebraic structures of an algebraic structure and other types of soft sets has been of interest by the researchers as Sezer, 2014;

Muştuoğlu et al., 2015; Ali et al., 2015; Sezer et al., 2015; Mahmood et al., 2015; Sezgin et al., 2017; Atagün and Sezgin, 2018; Sezgin, 2018; Mahmood et al., 2018; İftikhar and Mahmood, 2018; Jana et al., 2019; Mahmood, 2020; Özlü and Sezgin, 2020; Sezgin et al., 2022. Soft set theory and fuzzy set theory in different aspects have both theoretical and application aspects and they have been applied to decision making problems and real-life problems succesfully as Özer, 2022; Özlü, 2023a, 2023b, 2024, Özlü et al., 2024.

In the scope of algebra, one of the most important mathematical issues is to analyze the properties of the operation defined on a set to classify algebraic structures. In this study, we define a novel type of soft set operation called complementary extended lambda operation and we discuss its properties to contribute to the theory of soft set literature theoretically. In order to determine the relationship between the complementary exteneded lambda operation and other soft set operations, the distribution of complementary extended lambda operations over other kinds of soft set operations such as; restricted soft set operations, extended soft set operations and soft binary piecewise operations are examined and many interesting results have been obtained.

2. Preliminaries

Definition 2.1. Let E be the parameter set, U be the universal set, P(U) be the power set of U, and $M \subseteq E$. A pair (F, M) is called a soft set on U. Here, F is a function given by $F: M \rightarrow P(U)$ (Molodtsov, 1999).

$S_E(U)$ denotes the set of all soft sets over U throughout this paper. Let M be a fixed subset of E, then the set of all soft sets over U with M is indicated by $S_M(U)$. In other words, in the collection $S_M(U)$, only soft sets with the parameter set M are included, while in the collection $S_E(U)$, soft sets over U with any parameter set can be included.

Definition 2.2. Let (F, M) be a soft set over U. If $F(v)=\emptyset$ for all $v \in M$, then the soft set (F, M) is called a null soft set with respect to M, indicated by $\emptyset_M$. If for all $v \in M$, F(v)=U, then the soft set (F, M) is called a whole soft set with respect to M, indicated by U_M. The relative whole soft set U_E with respect to E is called the absolute soft set over U (Ali et al. 2009). A soft set with an empty parameter set is indicated by $\emptyset_\emptyset$, called by empty soft set, and $\emptyset_\emptyset$ is the only soft set with an empty parameter set (Ali et al., 2011)

Definition 2.3. For two soft sets (F, M) and (G, Y), we say that (F, M) is a soft subset of (G, Y) and it is indicated by $(F, M) \tilde{\subseteq} (G, Y)$, if $M \subseteq Y$ and $F(v) \subseteq G(v)$, for all $v \in M$. Two soft sets (F, M) and (G, Y) are said to be softequal if (F, M) is a soft subset of (G, Y) and (G, Y) is a soft subset of (F, M) (Pei and Miao, 2005).

Definition 2.4. The relative complement of a soft set (F, M), indicated by $(F, M)^r$, is defined by $(F, M)^r = (F^r, M)$, where $F^r: M \to P(U)$ is a mapping given by $(F, M)^r = U \backslash F(v)$ for all $v \in M$ (Ali et al. 2009). From now on, $U \backslash F(v)=[F(v)]'$ will be designated by $F'(v)$ for the sake of designation.

Çağman (2021) defined two new complements as inclusive and exclusive complements. + and θ denote inclusive and exclusive complements, respectively, and M and N are two sets, these binary operations, $M+N=M' \cup N$, $M\theta N=M' \cap N'$. Sezgin et al. (2023c) analyzed the relations between these two operations and also defined three new binary operations and examined their relations with each other. Let M and N be two sets, then $M*N=M' \cup N'$, $M\gamma N= M' \cap N$, $M \lambda N=M \cup N'$

Let $\circledast$ denote $\cap, \cup, -, \Delta, \lambda, \gamma, \theta, +, *$. Then, all the types of soft set operations may be given with the following generalised definitions:

Definition 2.5. Let $(F, M), (G, Y) \in S_E(U)$. The restricted $\circledast$ operation of (F,M) and (G, Y) is the soft set (H, Z), denoted to be $(F, M) \circledast_R (G, Y) = (H, Z)$, where $Z=M \cap Y \neq \emptyset$ and for all $v \in Z$, $H(v) = F(v) \circledast G(v)$. Here, if $Z= M \cap Y = \emptyset$, then $(F, M) \circledast_R (G, Y)=\emptyset_\emptyset$ (Ali et al., 2009; Ali et al, 2011; Sezgin and Atagün, 2011; Aybek, 2024).

Definition 2.6. Let $(F, M), (G, Y) \in S_E(U)$. The extended $\circledast$ operation (F, M) and (G, Y) is the soft set (H, Z), indicated by $(F, M) \circledast_\varepsilon (G, Y) = (H, Z)$, where $Z = M \cup Y$ and for all $v \in Z$,

$$H(v) = \begin{cases} F(v), & v \in M - Y \\ G(v), & v \in Y - M \\ F(v) \circledast G(v), & v \in M \cap Y \end{cases}$$

(Maji et al., 2003; Ali et al., 2009; Sezgin et al., 2019; Stojanavic, 2021; Aybek, 2024).

Definition 2.7. Let $(F, M), (G, Y) \in S_E(U)$. The complementary extended $\circledast$ operation (F, M) and (G, Y) is the soft set (H, Z), indicated by $(F, M) \overset{*}{\circledast}_\varepsilon (G, Y) = (H, Z)$, where $Z = M \cup Y$ and for all $v \in Z$,

$$H(v) = \begin{cases} F'(v), & v \in M - Y \\ G'(v), & v \in Y - M \\ F(v) \circledast G(v), & v \in M \cap Y \end{cases}$$

(Sarıalioğlu, 2024; Akbulut, 2024; Demirci, 2024).

Definition 2.8. Let $(F,M),(G,Y) \in S_E(U)$. The soft binary piecewise ⊛ of (F, M) and (G,Y) is the soft set (H,V), indicated by $(F,M)\underset{\circledast}{\sim}(G,Y) = (H,M)$, where for all v∈ M

$$H(v) = \begin{cases} F(v), & v \in M - Y \\ F(v) \circledast G(v), & v \in M \cap Y \end{cases}$$

(Eren, 2019; Sezgin and Yavuz, 2023b; Yavuz, 2024; Sezgin and Çalışıcı, 2024,).

Definition 2.9. Let $(F,M),(G,Y) \in S_E(U)$. The complemetary soft binary piecewise ⊛ of (F, M) and (G,Y) is the soft set (H,M), indicated by $(F,M)\overset{*}{\underset{\circledast}{\sim}}(G,Y) = (H,M)$, where for all v∈M

$$H(v) = \begin{cases} F'(v), & v \in M - Y \\ F(v) \circledast G(v), & v \in M \cap Y \end{cases}$$

(Sezgin and Demirci, 2023; Sezgin and Sarıalioğlu, 2024; Sezgin and Atagün, 2023; Sezgin and Aybek, 2023; Sezgin and Dagtoros, 2023; Sezgin et al. 2023a, 2023b; Sezgin and Yavuz, 2023a; Sezgin and Çağman, 2024).

For the possible future graph applications and network analysis as regards soft sets, we refer to Pant et al. (2024) which is motivated by the divisibility of determinants.

3. Complementary Extended Lambda Operation

In this section, complementary extended lambda operation is introduced, and its full algebraic properties are analyzed in detail.

Definition 3.1. Let (F, T) and (G, Ƶ) be two soft sets over U. The complementary extended lambda operation of (F, T) and (G, Ƶ) is the soft set (H, C), indicated by $(F,T)\overset{*}{\lambda_\varepsilon}(G,Ƶ)=(H,C)$, where for all ɰ∈C=T∪Ƶ;

$$H(ɰ)=\begin{cases} F'(ɰ) & ɰ\in T\setminus Ƶ \\ G'(ɰ) & ɰ\in Ƶ\setminus T \\ F(ɰ)\cup G'(ɰ) & ɰ\in T\cap Ƶ \end{cases}$$

Example 3.2. Let $E=\{e_1,e_2,e_3,e_4\}$ be the parameter set and $Ƶ=\{e_1, e_3\}$ and $B=\{e_2, e_3, e_4\}$ be two subsets of E and $U=\{h_1,h_2,h_3,h_4,h_5\}$ be the universal set. Assume that $(F, Ƶ)=\{(e_1,\{h_2,h_5\}),(e_3,\{h_1,h_2,h_5\})\}$, $(G,B)=\{(e_2,\{h_1,h_4,h_5\}),(e_3,\{h_2,h_3,h_4\}),(e_4,\{h_3,h_5\})\}$ be soft sets over U. Let $(F,T)\overset{*}{\lambda_\varepsilon}(G,Ƶ)=(H,T\cup Ƶ)$, where for all ɰ ∈ T∪Ƶ;

$$H(ɰ)=\begin{cases} F'(ɰ) & ɰ\in T\setminus Ƶ \\ G'(ɰ) & ɰ\in Ƶ\setminus T \\ F(ɰ)\cup G'(ɰ) & ɰ\in T\cap Ƶ \end{cases}$$

Here, since $T\cup Ƶ=\{e_1,e_2,e_3,e_4\}$, $T\setminus Ƶ=\{e_1\}$, $Ƶ\setminus B=\{e_2,e_4\}$, $T\cap Ƶ=\{e_3\}$, $H(e_1)=F'(e_1)=\{h_1,h_3,h_4\}$, $H(e_2)=G'(e_2)=\{h_2,h_3\}$, $H(e_4)=G'(e_4)=\{h_1,h_2,h_4\}$ and $H(e_3)=F(e_3)\cup G'(e_3)=\{h_1,h_2,h_5\}$.

Thus, $(F,T)\overset{*}{\lambda}_{\varepsilon}(G,Ƶ)=\{(e_1,\{h_1,h_3,h_4\}),(e_2,\{h_2,h_3\}),(e_3,\{h_1,h_2,h_5\}),(e_4,\{h_1,h_2,h_4\})\}$.

Proposition 3.3. $\overset{*}{\lambda}_{\varepsilon}$ is closed in $S_E(U)$.

Proof: $\overset{*}{\lambda}_{\varepsilon}: S_E(U)\times S_E(U)\rightarrow S_E(U)$

$$((F,T),(G,Ƶ))\rightarrow (F,T)\overset{*}{\lambda}_{\varepsilon}(G,Ƶ)=(H,T\cup Ƶ)$$

Similarly,

$$\overset{*}{\lambda}_{\varepsilon}: S_T(U)\times S_T(U)\rightarrow S_T(U)$$

$$((F,T),(G,T))\rightarrow (F,T)\overset{*}{\lambda}_{\varepsilon}(G,T)=(K,T\cup T)=(K,T)$$

That is, when T is a fixed subset of the set E and (F, T) and (G, T) are elements of $S_T(U)$, then so is $(F,T)\overset{*}{\lambda}_{\varepsilon}(G,T)$. Namely, S_T is closed under $\overset{*}{\lambda}_{\varepsilon}$ as well.

Proposition 3.4. $[(F,T)\overset{*}{\lambda}_{\varepsilon}(G,Ƶ)]\overset{*}{\lambda}_{\varepsilon}(H,M)\neq (F,T)\overset{*}{\lambda}_{\varepsilon}[(G,Ƶ)\overset{*}{\lambda}_{\varepsilon}(H,M)]$.

Proof: Firstly, let's consider the left hand side (LHS). Suppose $(F,T)\overset{*}{\lambda}_{\varepsilon}(G,Ƶ)=(S,T\cup Ƶ)$, where for all $ɰ\in T\cup Ƶ$;

$$S(ɰ)=\begin{cases} F'(ɰ) & ɰ\in T\setminus Ƶ \\ G'(ɰ) & ɰ\in Ƶ\setminus T \\ F(ɰ)\cup G'(ɰ) & ɰ\in T\cap Ƶ \end{cases}$$

Let $(S,T\cup Ƶ)\overset{*}{\lambda}_{\varepsilon}(H,M)=(R,(T\cup Ƶ)\cup M))$, where for all $ɰ\in(T\cup Ƶ)\cup M$,

$$R(ɰ)=\begin{cases} S'(ɰ) & ɰ\in (T\cup Ƶ)\setminus M \\ H'(ɰ) & ɰ\in M\setminus(T\cup M) \\ S(ɰ)\cup H'(ɰ) & ɰ\in (T\cup Ƶ)\cap M \end{cases}$$

Hence,

$$R(ɰ)=\begin{cases} F(ɰ) & ɰ\in(T\setminus Ƶ)\setminus M=T\cap Ƶ'\cap M' \\ G(ɰ) & ɰ\in(Ƶ\setminus T)\setminus M=T'\cap Ƶ\cap M' \\ F'(ɰ)\cap G(ɰ) & ɰ\in(T\cap Ƶ)\setminus M=T\cap Ƶ\cap M' \\ H'(ɰ) & ɰ\in M\setminus(T\cup Ƶ)=T'\cap Ƶ'\cap M \\ F'(ɰ)\cup H'(ɰ) & ɰ\in(T\setminus Ƶ)\cap M=T\cap Ƶ'\cap M \\ G'(ɰ)\cup H'(ɰ) & ɰ\in(Ƶ\setminus T)\cap M=T'\cap Ƶ\cap M \\ [F'(ɰ)\cap G(ɰ)]\cup H'(ɰ) & ɰ\in(T\cap Ƶ)\cap M=T\cap Ƶ\cap M \end{cases}$$

$(G, Ƶ)\ \overset{*}{\lambda_\varepsilon}(H,M)=(L,Ƶ\cup M)$, where for all $ɰ\in Ƶ\cup M$;

$$L(ɰ)=\begin{cases} G'(ɰ) & ɰ\in Ƶ\setminus M \\ H'(ɰ) & ɰ\in M\setminus Ƶ \\ G(ɰ)\cup H'(ɰ) & ɰ\in Ƶ\cap M \end{cases}$$

$(F, T)\ \overset{*}{\lambda_\varepsilon}(L,Ƶ\cup M)=(N,(T\cup(Ƶ\cup M))$, where for all $ɰ\in T\cup Ƶ\cup M$;

$$N(ɰ)=\begin{cases} F'(ɰ) & ɰ\in T\setminus(Ƶ\cup M) \\ R'(ɰ) & ɰ\in(Ƶ\cup M)\setminus T \\ F(ɰ)\cup R'(ɰ) & ɰ\in T\cap(Ƶ\cup M) \end{cases}$$

Hence,

$$N(ɰ)=\begin{cases} F'(ɰ) & ɰ\in T\setminus(Ƶ\cup M)=T\cap Ƶ'\cap M' \\ G(ɰ) & ɰ\in(Ƶ\setminus M)\setminus T=T'\cap Ƶ\cap M' \\ H(ɰ) & ɰ\in(M\setminus Ƶ)\setminus T=T'\cap Ƶ'\cap M \\ G'(ɰ)\cap H(ɰ) & ɰ\in(Ƶ\cap M)\setminus T=T'\cap Ƶ\cap M \\ F(ɰ)\cup G(ɰ) & ɰ\in T\cap(Ƶ\setminus M)=T\cap Ƶ\cap M' \\ F(ɰ)\cup H(ɰ) & ɰ\in T\cap(M\setminus Ƶ)=T\cap Ƶ'\cap M \\ F(ɰ)\cup[G'(ɰ)\cap H(ɰ)] & ɰ\in T\cap(Ƶ\cap M)=T\cap Ƶ\cap M \end{cases}$$

Thus, $(R, (T\cup Ƶ)\cup M)\neq(N,T\cup(Ƶ\cup M))$. That is, in the set $S_E(U)$, $\overset{*}{\lambda_\varepsilon}$ is not associative.

Moreover, we have the following:

Proposition 3.5. $[(F, T)\ \overset{*}{\lambda_\varepsilon}(G, T)]\ \overset{*}{\lambda_\varepsilon}(H,T)\neq(F,T)\ \overset{*}{\lambda_\varepsilon}[(G,T)\ \overset{*}{\lambda_\varepsilon}(H,T)]$.

Proof: Since $[F'(ɰ)\cap G(ɰ)]\cup H'(ɰ)\neq F(ɰ)\cup[G'(ɰ)\cap H(ɰ)]$, in the set $S_T(U)$, $\overset{*}{\lambda_\varepsilon}$ is not associative

Proposition 3.6. $(F, T)\ \overset{*}{\lambda_\varepsilon}(G,Ƶ)\neq(G,Ƶ)\ \overset{*}{\lambda_\varepsilon}(F,T)$.

Proof: Firstly, the parameter sets of the soft set on both sides of the equation is T∪Ƶ, and thus the first condition of the soft equality is satisfied. Now let's consider the LHS. Let (F, T) $\overset{*}{\lambda}_{\varepsilon}$(G,Ƶ)=(H,T∪Ƶ), where for all ɰ∈T∪Ƶ ,

H(ɰ)= { F'(ɰ) ɰ∈T\Ƶ ; G'(ɰ) ɰ∈Ƶ\T ; F(ɰ)∪G'(ɰ) ɰ∈T∩Ƶ

Now consider the RHS. Let (G,Ƶ) $\overset{*}{\lambda}_{\varepsilon}$(F,T)=(S,Ƶ∪T), where for all ɰ∈Ƶ∪T ,

S(ɰ) = { G'(ɰ) ɰ∈Ƶ\T ; F'(ɰ) ɰ∈T\Ƶ ; G(ɰ)∪F'(ɰ) ɰ∈Ƶ∩T

Hence, (F,T) $\overset{*}{\lambda}_{\varepsilon}$(G,Ƶ)≠(G,Ƶ) $\overset{*}{\lambda}_{\varepsilon}$(F,T). But, if Ƶ∩T=∅, then (F,T) $\overset{*}{\lambda}_{\varepsilon}$ (G,Ƶ)=(G,Ƶ) $\overset{*}{\lambda}_{\varepsilon}$(F,T), Moreover, (F,T) $\overset{*}{\lambda}_{\varepsilon}$(G,T)≠(G,T) $\overset{*}{\lambda}_{\varepsilon}$(F,T), Hence, in S_E(U) and S_T(U), $\overset{*}{\lambda}_{\varepsilon}$ is not commutative.

Proposition 3.7. (F,T) $\overset{*}{\lambda}_{\varepsilon}$ (F,T)= U_T

Proof: Let (F,T) $\overset{*}{\backslash}_{\varepsilon}$(F,T)=(H,T). Hence, for all ɰ∈T, H(ɰ)=F(ɰ)∪F'(ɰ)=U, thus (H,T)=U_T. That is, $\overset{*}{\lambda}_{\varepsilon}$ is not idempotent in S_E(U).

Proposition 3.8. (F,T) $\overset{*}{\lambda}_{\varepsilon}$ $\emptyset_T$=U_T

Proof: Let $\emptyset_T$=(S,T). Thus, for all ɰ∈T, S(ɰ)= ∅. Let (F,T) $\overset{*}{\lambda}_{\varepsilon}$(S,T)=(H,T), where for all ɰ∈T; H(ɰ)=F(ɰ)∪S'(ɰ)=F(ɰ)∪ U=U. Thus, (H,T)= U_T .

Proposition 3.9. $\emptyset_T$ $\overset{*}{\lambda}_{\varepsilon}$(F,T)=(F,T)^r

Proof: Let $\emptyset_T$=(S,T). Hence, for all ɰ∈T, S(ɰ)=∅. Let (S,T) $\overset{*}{\lambda}_{\varepsilon}$(F,T)=(H,T), where for all ɰ∈T; H(ɰ)=S(ɰ)∪F'(ɰ)=∅ ∪F'(ɰ)=F'(ɰ). Thus, (H,T)= (F,T)^r.

Proposition 3.10. (F,T) $\overset{*}{\lambda}_{\varepsilon}$ $\emptyset_\emptyset$= $\emptyset_\emptyset$ $\overset{*}{\lambda}_{\varepsilon}$(F,T)= (F,T)^r

Proof: Let $\emptyset_\emptyset$=(S,∅) and (F,T) $\overset{*}{\lambda}_{\varepsilon}$(S, ∅)=(H,T∪∅), where for all ɰ∈T∪∅=T,

H(ɰ)= { F'(ɰ) ɰ∈T\∅=T ; S'(ɰ) ɰ∈∅ \T=∅ ; F(ɰ)∪S'(ɰ) ɰ∈T∩∅=∅

Thus, for all ɰ∈T, H(ɰ)=F'(ɰ), and so (H,T)=(F,T)^r. Similarly, let (S,∅) $\overset{*}{\lambda_\varepsilon}$(F,T)=(K, ∅∪T), where for all ɰ∈∅∪T=T;

$$K(ɰ)=\begin{cases} S'(ɰ) & ɰ\in\emptyset\setminus T=\emptyset \\ F'(ɰ) & ɰ\in T\setminus\emptyset=T \\ S(ɰ)\cup F'(ɰ) & ɰ\in\emptyset\cap T=\emptyset \end{cases}$$

Thus, for all ɰ∈T, K(ɰ)=F'(ɰ), (K,T)=(F,T)^r

Proposition 3.11. (F,T) $\overset{*}{\lambda_\varepsilon}\emptyset_E = U_E$

***Proof*:**: Let $\emptyset_E$=(T,E). Thus, for all ɰ∈E, T(ɰ)=∅. Let (F,T) $\overset{*}{\lambda_\varepsilon}$(T,E)=(H,T∪E), where for all ɰ∈T∪E=E;

$$H(ɰ)=\begin{cases} F'(ɰ) & ɰ\in T\setminus E=\emptyset \\ T'(ɰ) & ɰ\in E\setminus T=T' \\ F(ɰ)\cup T'(ɰ) & ɰ\in T\cap E=T \end{cases}$$

Thus,

$$H(ɰ)=\begin{cases} F'(ɰ) & ɰ\in T\setminus E=\emptyset \\ U & ɰ\in E\setminus T=T' \\ U & ɰ\in T\cap E=T \end{cases}$$

For all ɰ∈E; H(ɰ)=U, so (H,E)= U_E.

Proposition 3.12. (F,T) $\overset{*}{\lambda_\varepsilon}U_T$=(F,T)

Proof: Let U_T=(K,T). Thus, for all ɰ∈T, K(ɰ)=U. Let (F,T) $\overset{*}{\lambda_\varepsilon}$(K, T)=(H,T). Hence, for all ɰ∈T; H(ɰ)= F(ɰ)∪T'(ɰ)=F(ɰ)∪ ∅=F(ɰ).Thus, (H,T)=(F,T).

That is, in S_T(U), the right identity element of $\overset{*}{\lambda_\varepsilon}$ is the soft set U_T.

Proposition 3.13. U_T $\overset{*}{\lambda_\varepsilon}$(F,T)= U_T

Proof: Let U_T=(K,T). Thus, for all ɰ∈T; K(ɰ)=U. Let (K,T) $\overset{*}{\lambda_\varepsilon}$(F,T)=(H,T), where for all ɰ∈T; H(ɰ)=T(ɰ)∪F'(ɰ)=U∪F'(ɰ)=U_T. Thus, (H,T)= U_T. That is, the left absorbing element of $\overset{*}{\lambda_\varepsilon}$ in S_T(U) is the soft set U_T .

Proposition 3.14. (F,T) $\overset{*}{\lambda_\varepsilon}$ (F,T)^r=(F,T).

Proof: Let $(F,T)^r=(H,T)$. Thus, for all ɰ∈T; H(ɰ)=F'(ɰ). Let $(F,T)\overset{*}{\lambda}_{\varepsilon}(H,T)=(L,T)$, where for all ɰ∈T; L(ɰ)=F(ɰ)∪H'(ɰ)=F(ɰ)∪F(ɰ)=F(ɰ).Thus, (L,T)=(F,T).

That is, in $S_E(U)$, the complement of every element is its own right identity for $\overset{*}{\lambda}_{\varepsilon}$.

Proposition 3.15. $(F,T)^r\overset{*}{\lambda}_{\varepsilon}(F,T)=(F,T)^r$.

Proof: Let $(F,T)^r=(H,T)$. Thus, for all ɰ∈T, H(ɰ)=F'(ɰ). Let $(H,T)\overset{*}{\lambda}_{\varepsilon}(F,T)=(L,T)$, where for all ɰ∈T, T(ɰ)=H(ɰ)∪F'(ɰ)=F'(ɰ)∪F'(ɰ)=F'(ɰ). Thus $(L,T)=(F,T)^r$.

That is, in $S_E(U)$, the complement of every element is its own left absorbing element for $\overset{*}{\lambda}_{\varepsilon}$.

Proposition 3.16. $[(F,T)\overset{*}{\lambda}_{\varepsilon}(G,Ƶ)]^r=(F,T)\,\gamma_{\varepsilon}\,(G,Ƶ)$.

Proof: Let $(F,T)\overset{*}{\backslash}_{\varepsilon}(G,Ƶ)=(H,T\cup Ƶ)$, where for all ɰ∈T∪ Ƶ;

$$H(ɰ)=\begin{cases} F'(ɰ) & ɰ\in T\backslash Ƶ \\ G'(ɰ) & ɰ\in Ƶ\backslash T \\ F(ɰ)\cup G'(ɰ) & ɰ\in T\cap Ƶ \end{cases}$$

Let $(H,T\cup Ƶ)^r=(K,T\cup Ƶ)$, where for all ɰ∈T∪ Ƶ;

$$K(ɰ)=\begin{cases} F(ɰ) & ɰ\in T\backslash Ƶ \\ G(ɰ) & ɰ\in Ƶ\backslash T \\ F'(ɰ)\cap G(ɰ) & ɰ\in T\cap Ƶ \end{cases}$$

Thus, $(K,T\cup Ƶ)=(F,T)+_{\varepsilon}(G,Ƶ)$.

Proposition 3.17. $(F,T)\overset{*}{\lambda}_{\varepsilon}(G,T)=\emptyset_T \Leftrightarrow (F,T)=\emptyset_T$ and $(G,T)=U_T$.

Proof: Let $(F,T)\overset{*}{\lambda}_{\varepsilon}(G,T)=(K,T)$, where for all ɰ∈T; K(ɰ)=F(ɰ)∪G'(ɰ). Since $(K,T)=\emptyset_T$, for all ɰ∈T, K(ɰ)=∅. Thus, for all ɰ∈T; K(ɰ)=F(ɰ)∪G'(ɰ)=∅ ⇔ for all ω ∈ T, F(ɰ)=∅ and G'(ɰ)=∅ ⇔ for all ɰ∈T, F(ɰ)=∅ and G(ɰ)=U ⇔$(F,T)=\emptyset_T$ and $(G,T)=U_T$

Proposition 3.18. $\emptyset_T \,\tilde{\subseteq}(F,T)\overset{*}{\lambda}_{\varepsilon}(G,Ƶ)$, $\emptyset_Z \,\tilde{\subseteq}(F,T)\overset{*}{\lambda}_{\varepsilon}(G,Ƶ)$, $\emptyset_Z \,\tilde{\subseteq}(G,Ƶ)\overset{*}{\lambda}_{\varepsilon}(F,T)$, $\emptyset_T \,\tilde{\subseteq}(G,Ƶ)\overset{*}{\lambda}_{\varepsilon}(F,T)$. Besides, $(F,T)\overset{*}{\lambda}_{\varepsilon}(G,Ƶ)\,\tilde{\subseteq}\, U_{T\cup Z}$ and $(G,Ƶ)\overset{*}{\lambda}_{\varepsilon}(F,T)\,\tilde{\subseteq}\, U_{Z\cup T}$.

Proposition 3.19. $(F,T)\,\tilde{\subseteq}(F,T)\overset{*}{\lambda}_{\varepsilon}(G,T)$ and $(G,T)^r\,\tilde{\subseteq}(F,T)\overset{*}{\lambda}_{\varepsilon}(G,T)$

Proof: Let $(F,T)\overset{*}{\lambda}_{\varepsilon}(G,T)=(H,Ƶ)$, where for all ɰ∈T; H(ɰ)=F(ω) ∪G'(ɰ). Thus, for all ɰ∈T, H(ω)=

$F(\omega) \subseteq F(\omega) \cup G'(\omega)$ ve $H(\omega) = G'(\omega) \subseteq F(\omega) \cup G'(\omega)$. Thus, (F,T) $\tilde{\subseteq}$(F,T) $\overset{*}{\lambda_\varepsilon}$(G,T) and $(G,T)^r$ $\tilde{\subseteq}$(F,T) $\overset{*}{\lambda_\varepsilon}$(G,T)

Proposition 3.20. If (F,T) $\tilde{\subseteq}$ (G,T), then (H,Ƶ) $\overset{*}{\lambda_\varepsilon}$(G,T) $\tilde{\subseteq}$(H,Ƶ) $\overset{*}{\lambda_\varepsilon}$(F,T)

***Proof*:** Let (F,T)$\tilde{\subseteq}$ (G,T), where for all ɰ∈T, F(ɰ)⊆G(ɰ) and G'(ɰ)⊆F'(ɰ). Let (H,Ƶ) $\overset{*}{\lambda_\varepsilon}$(G,T)=(Y,Ƶ∪T), where for all ɰ∈Ƶ∪T;

$$Y(ɰ)=\begin{cases} H'(ɰ) & ɰ\in Ƶ\setminus T \\ G'(ɰ) & ɰ\in T\setminus Ƶ \\ H(ɰ)\cup G'(ɰ) & ɰ\in Ƶ\cap T \end{cases}$$

Let (H,Ƶ) $\overset{*}{\lambda_\varepsilon}$ (F ,T)=(W,Ƶ∪T), where for all ɰ∈Ƶ∪T;

$$W(ɰ)=\begin{cases} H'(ɰ) & ɰ\in Ƶ\setminus T \\ F'(ɰ) & ɰ\in T\setminus Ƶ \\ H(ɰ)\cup F'(ɰ) & ɰ\in Ƶ\cap T \end{cases}$$

If ɰ∈Ƶ\T; Y(ɰ)=H'(ɰ), then W(ɰ)=H'(ɰ), and so Y(ɰ)= $H'(\omega) \subseteq$ H'(ɰ)=W(ɰ); if ɰ∈T\Ƶ, Y(ɰ)=G'(ɰ), then W(ɰ)=F'(ɰ) and so Y(ɰ)= $G'(\omega) \subseteq$F'(ɰ)=W(ɰ); if ɰ∈T∩Ƶ, Y(ɰ)=H(ɰ)∪G'(ɰ) and so W(ɰ)=F(ɰ)∪H'(ɰ),Y(ɰ)= $H(\omega) \cup G'(\omega) \subseteq$H(ɰ)∪F'(ɰ)=W(ɰ). Thus, for all ɰ∈Ƶ∪T; Y(ɰ)⊆W(ɰ). Hence, (H,T) $\overset{*}{\lambda_\varepsilon}$(G,T) $\tilde{\subseteq}$(H,T) $\overset{*}{\lambda_\varepsilon}$(F,T).

Proposition 3.21. If (H,Ƶ) $\overset{*}{\lambda_\varepsilon}$(G,T) $\tilde{\subseteq}$(H,Ƶ) $\overset{*}{\lambda_\varepsilon}$(F,T), then (F,T) $\tilde{\subseteq}$ (G,T) needs not be true. That is the converse of Proposition 3.20 is not true in general.

Proof: Let E={e_1,e_2,e_3,e_4,e_5, e_6} be the parameter set, T={e_1,e_3}, Ƶ={e_1,e_3, e_5} be the subsets of E, U={h_1,h_2, h_3,h_4, h_5} be the universal set, and (F,T), (G,T) ve (H,Ƶ) be soft sets over U such that (F,T)={(e_1,{h_2, h_5}),(e_3,∅)}, (G,T)={(e_1,∅)},(e_3,∅)}, (H,Ƶ)={(e_1,U),(e_3,U),(e_5, ∅)}.

Let (H,Ƶ) $\overset{*}{\lambda_\varepsilon}$(G,T)=(L,Ƶ∪ T), where for all ω ∈ Ƶ ∪ T={e_1, e_3, e_5}, L(e_1)=H(e_1) ∪G'(e_1)=U, L(e_3)=H(e_3) ∪G'(e_3)=U, L(e_5)=H'(e_5)=U .

Thus, (H,Ƶ) $\overset{*}{\lambda_\varepsilon}$(G,T) ={($e_1$,U),($e_3$,U),($e_5$,U)}. Let (H,Ƶ) $\overset{*}{\lambda_\varepsilon}$(F,T)=(W,Ƶ ∪ T), where for all ω ∈ Ƶ ∪ T={e_1, e_3, e_5}, W(e_1)=H(e_1) ∪F'(e_1)=U, W(e_3)=H(e_3) ∪F'(e_3)=U,

$W(e_5)=H'(e_5)=U$. Thus, $(H,Ƶ)\overset{*}{\lambda_\varepsilon}(F,T)=\{(e_1,U), (e_3,U), (e_5,U)\}$. Thus, $(H,Ƶ)\overset{*}{\lambda_\varepsilon}(G,T) \cong\!\subseteq (H,Ƶ)\overset{*}{\lambda_\varepsilon}(F,T)$, but (F,T) is not a soft subset of (G, T).

Proposition 3.22. If $(F,T)\cong\!\subseteq(G,T)$ and $(K,T)\cong\!\subseteq(L,T)$, then $(F,T)\overset{*}{\lambda_\varepsilon}(L,T)\cong\!\subseteq(G,T)\overset{*}{\lambda_\varepsilon}(K,T)$ and $(K,T)\overset{*}{\lambda_\varepsilon}(G,T)\cong\!\subseteq(L,T)\overset{*}{\lambda_\varepsilon}(F,T)$.

Proof: Let $(F,T)\cong\!\subseteq(G,T)$ and $(K,T)\cong\!\subseteq(L,T)$. Thus, for all $\omega\in T$; $F(\omega)\subseteq G(\omega)$ and $K(\omega)\subseteq L(\omega)$. Thus, $G'(\omega)\subseteq F'(\omega)$ and $L'(\omega)\subseteq K'(\omega)$. Hence for all $\omega\in T$; $F(\omega)\cup L'(\omega)\subseteq G(\omega)\cup K'(\omega)$ and $K(\omega)\cup G'(\omega)\subseteq L(\omega)\cup F'(\omega)$.

4. Distributions of complementary extended lambda operations over other type of soft set operations

Theorem 4.1. Let $(F,T),(G,Ƶ),(H,M)$ be soft sets over U. The complementary extended lambda operation has the following distributions over restricted soft set operations

i) LHS Distributions

1)If $T\cap(Ƶ\Delta M)=\emptyset$, then $(F,T)\overset{*}{\lambda_\varepsilon}[(G,Ƶ)\cup_R(H,M)]=[(F,T)\overset{*}{\lambda_\varepsilon}(G,Ƶ)]\cap_R[(F,T)\overset{*}{\lambda_\varepsilon}(H,M)]$

Proof: Consider the LHS. Let $(G,Ƶ)\cup_R(H,M)=(M,Ƶ\cap M)$, where for all $\omega\in Ƶ\cap M$; $M(\omega)=G(\omega)\cup H(\omega)$. Let $(F,T)\overset{*}{\lambda_\varepsilon}(M,Ƶ\cap M)=(N,T\cup(Ƶ\cap M))$, where for all $\omega\in T\cup(Ƶ\cap M)$;

$$N(\omega)=\begin{cases} F'(\omega) & \omega\in T\setminus(Ƶ\cap M) \\ M'(\omega) & \omega\in(Ƶ\cap M)\setminus T \\ F(\omega)\cup M'(\omega) & \omega\in T\cap(Ƶ\cap M) \end{cases}$$

Hence,

$$N(\omega)=\begin{cases} F'(\omega) & \omega\in T\setminus(Ƶ\cap M) \\ G'(\omega)\cap H'(\omega) & \omega\in(Ƶ\cap M)\cap T' \\ F(\omega)\cup[G'(\omega)\cap H'(\omega)] & \omega\in T\cap(Ƶ\cap M) \end{cases}$$

Now consider the RHS. Let $(F,T)\overset{*}{\lambda_\varepsilon}(G,Ƶ)=(M,T\cup Ƶ)$, where for all $\omega\in T\cup Ƶ$;

$$M(\omega)=\begin{cases} F'(\omega) & \omega\in T\setminus Ƶ \\ G'(\omega) & \omega\in Ƶ\setminus T \\ F(\omega)\cup G'(\omega) & \omega\in T\cap Ƶ \end{cases}$$

Let $(F,T)\ \overset{*}{\lambda}_{\varepsilon}(H,M)=(K,T\cup M)$, where for all $\omega\in T\cup M$;

$$K(\omega)=\begin{cases} F'(\omega) & \omega\in T\setminus M \\ H'(\omega) & \omega\in M\setminus T \\ F(\omega)\cup H'(\omega) & \omega\in T\cap M \end{cases}$$

Let $(M,T\cup Ƶ)\cap_R(K,T\cup M)=(W,(T\cup Ƶ)\cap(T\cup M))$, where for all $\omega\in(T\cup Ƶ)\cap(T\cup M)$; $W(\omega)=T(\omega)\cap K(\omega)$. Thus,

$$W(\omega)=\begin{cases} F'(\omega)\cap F'(\omega) & \omega\in(T\setminus Ƶ)\cap(T\setminus M)=T\cap Ƶ'\cap M' \\ F'(\omega)\cap H'(\omega) & \omega\in(T\setminus Ƶ)\cap(M\setminus T)=\emptyset \\ F'(\omega)\cap[F(\omega)\cup H'(\omega)] & \omega\in(T\setminus Ƶ)\cap(T\cap M)=T\cap Ƶ'\cap M \\ G'(\omega)\cap F'(\omega) & \omega\in(Ƶ\setminus T)\cap(T\setminus M)=\emptyset \\ G'(\omega)\cap H'(\omega) & \omega\in(Ƶ\setminus T)\cap(M\setminus T)=T'\cap Ƶ\cap M \\ G'(\omega)\cap[F(\omega)\cup H'(\omega)] & \omega\in(Ƶ\setminus T)\cap(T\cap M)=\emptyset \\ [F(\omega)\cup G'(\omega)]\cap F'(\omega) & \omega\in(T\cap Ƶ)\cap(T\setminus M)=T\cap Ƶ\cap M' \\ [F(\omega)\cup G'(\omega)]\cap H'(\omega) & \omega\in(T\cap Ƶ)\cap(M\setminus T)=\emptyset \\ [F(\omega)\cup G'(\omega)]\cap[F(\omega)\cup H'(\omega)] & \omega\in(T\cap Ƶ)\cap(T\cap M)=T\cap Ƶ\cap M \end{cases}$$

Hence,

$$W(\omega)=\begin{cases} F'(\omega) & \omega\in T\cap Ƶ'\cap M' \\ F'(\omega)\cap H'(\omega) & \omega\in T\cap Ƶ'\cap M \\ G'(\omega)\cap H'(\omega) & \omega\in T'\cap Ƶ\cap M \\ G'(\omega)\cap F'(\omega) & \omega\in T\cap Ƶ\cap M' \\ [F(\omega)\cup G'(\omega)]\cap[F(\omega)\cup H'(\omega)] & \omega\in T\cap Ƶ\cap M \end{cases}$$

Here, when considering the $T\setminus(Ƶ\cap M)$ in the function N, since $T\setminus(Ƶ\cap M)=T\setminus(Ƶ\cap M)'$, if an element is in the complement of $(Ƶ\cap M)$, it is either in $Ƶ\setminus M$, in $M\setminus Ƶ$, or in $(Ƶ\cup M)'$. Thus, if $\omega\in T\setminus(Ƶ\cap M$, then $\omega\in T\cap Ƶ\cap M'$or $\omega\in T\cap Ƶ'\cap M$ or $\omega\in T\cap Ƶ'\cap M'$. Thus, N=T under the conditions $T\cap Ƶ'\cap M=T\cap Ƶ\cap M'=\emptyset$. It is obvious that the condition $T\cap Ƶ'\cap M=T\cap Ƶ\cap M'=\emptyset$ is equivalent to the condition $T\cap(Z\Delta M)=\emptyset$.

2) If $T\cap(Z\Delta M)=\emptyset$, then $(F,T)\ \overset{*}{\lambda}_{\varepsilon}[(G,Ƶ)\ \cap_R(H,M)]=[(F,T)\ \overset{*}{\lambda}_{\varepsilon}(G,Ƶ)]\ \cup_R[(F,T)\ \overset{*}{\lambda}_{\varepsilon}(H,M)]$

ii) RHS Distributions

1)If $\cap Ƶ\cap M=\emptyset$, then $[(F,T)\cup_R(G,Ƶ)]\ \overset{*}{\lambda}_{\varepsilon}(H,M)=[(F,T)\ \overset{*}{\lambda}_{\varepsilon}(H,M)]\ \cap_R\ [(G,Ƶ)\ \overset{*}{\lambda}_{\varepsilon}(H,M)]$.

Proof: Consider the LHS. Let $(F,T)\cup_R(G,Ƶ)=(R,T\cap Ƶ)$, where for all $\omega\in T\cap Ƶ$; $R(\omega)=F(\omega)\cup G(\omega)$. Let $(R,T\cap Ƶ)\ \overset{*}{\lambda_\varepsilon}(H,M)=(L,(T\cap Ƶ)\cup M)$, where for all $\omega\in(T\cap Ƶ)\cup M$;

$$L(\omega)=\begin{cases} R'(\omega) & \omega\in(T\cap Ƶ)\setminus M \\ H'(\omega) & \omega\in M\setminus(T\cap Ƶ) \\ R(\omega)\cup H'(\omega) & \omega\in(T\cap Ƶ)\cap M \end{cases}$$

Hence,

$$L(\omega)=\begin{cases} F'(\omega)\cap G'(\omega) & \omega\in(T\cap Ƶ)\setminus M \\ H'(\omega) & \omega\in M\setminus(T\cap Ƶ) \\ [F(\omega)\cup G(\omega)]\cup H'(\omega) & \omega\in(T\cap Ƶ)\cap M \end{cases}$$

Now consider the RHS, i.e, $[(F,T)\ \overset{*}{\lambda_\varepsilon}(H,M)]\cap_R[(G,Ƶ)\ \overset{*}{\lambda_\varepsilon}(H,M)]$ Let $(F,T)\ \overset{*}{\lambda_\varepsilon}(H,M)=(S,T\cup M)$, where for all $\omega\in T\cup M$;

$$S(\omega)=\begin{cases} F'(\omega) & \omega\in T\setminus M \\ H'(\omega) & \omega\in M\setminus T \\ F(\omega)\cup H'(\omega) & \omega\in T\cap M \end{cases}$$

Let $(G,Ƶ)\ \overset{*}{\lambda_\varepsilon}(H,M)=(K,Ƶ\cup M)$, where for all $\omega\in Ƶ\cup M$;

$$K(\omega)=\begin{cases} G'(\omega) & \omega\in Ƶ\setminus M \\ H'(\omega & \omega\in M\setminus Ƶ \\ G(\omega)\cup H'(\omega) & \omega\in Ƶ\cap M \end{cases}$$

Let $(S,T\cup Ƶ)\cap_R(K,Ƶ\cup M)=(W,(T\cup Ƶ)\cap(Ƶ\cup M))$, where for all $\omega\in(T\cup Ƶ)\cap(Ƶ\cup M)$; $W(\omega)=S(\omega)\cap K(\omega)$. Hence,

$$W(\omega)=\begin{cases} F'(\omega)\cap G'(\omega) & \omega\in(T\setminus M)\cap(Ƶ\setminus M)=T\cap Ƶ\cap M' \\ F'(\omega)\cap H'(\omega) & \omega\in(T\setminus M)\cap(M\setminus Ƶ)=\emptyset \\ F'(\omega)\cap[G(\omega)\cup H'(\omega)] & \omega\in(T\setminus M)\cap(Ƶ\cap M)=\emptyset \\ H'(\omega)\cap G'(\omega) & \omega\in(M\setminus T)\cap(Ƶ\setminus M)=\emptyset \\ H'(\omega)\cap H'(\omega) & \omega\in(M\setminus T)\cap(M\setminus Ƶ)=T'\cap Ƶ'\cap M \\ H'(\omega)\cap[G(\omega)\cup H('\omega)] & \omega\in(M\setminus T)\cap(Ƶ\cap M)=T'\cap Ƶ\cap M \\ [F(\omega)\cup H'(\omega)]\cap G'(\omega) & \omega\in(T\cap M)\cap(Ƶ\setminus M)=\emptyset \\ [F(\omega)\cup H'(\omega)]\cap H'(\omega) & \omega\in(T\cap M)\cap(M\setminus Ƶ)=T\cap Ƶ'\cap M \\ [F(\omega)\cup H'(\omega)]\cap[G(\omega)\cup H'(\omega)] & \omega\in(T\cap M)\cap(Ƶ\cap M)=T\cap Ƶ\cap M \end{cases}$$

Thus,

$$W(\omega)=\begin{cases} F'(\omega)\cap G'(\omega) & \omega\in T\cap Ƶ\cap M' \\ H'(\omega) & \omega\in T'\cap Ƶ'\cap M \\ H'(\omega) & \omega\in T'\cap Ƶ\cap M \\ H'(\omega) & \omega\in T\cap Ƶ'\cap M \\ [F(\omega)\cap G(\omega)]\cup H'(\omega) & \omega\in T\cap Ƶ\cap M \end{cases}$$

Here, when considering the M\(T∩Ƶ) in the function L, since M\(T∩Ƶ)=M∩(T∩Ƶ)', if an element is in the complement of (T∩Ƶ), it is either in T\Ƶ, Ƶ\B or in (T∪Ƶ)'. Thus, if $\omega\in M\setminus(T\cap Ƶ)$, then ω∈M∩T∩Ƶ' or ω∈ M∩Ƶ∩T' or ω∈ M∩T'∩Ƶ'. Hence, L=W is satisfied if T∩Ƶ∩M=∅

2) If T∩Ƶ∩M'=T∩Ƶ∩M=∅, then $[(F,T)\cap_R(G,Ƶ)]\overset{*}{\lambda_\varepsilon}(H,M)=[(F,T)\overset{*}{\lambda_\varepsilon}(H,M)]\cap_R[(G,Ƶ)\overset{*}{\lambda_\varepsilon}(H,M)]$.

Theorem 4.2. Let $(F,T),(G,Ƶ),(H,M)$ be soft sets over U. Then, the following distributions of the complementary extended lambda operation over extended soft set operations hold:

i) LHS Distributions

1)If T∩(ƵΔM)=∅, then $(F,T)\overset{*}{\lambda_\varepsilon}[(G,Ƶ)\cap_\varepsilon(H,M)]=[(F,T)\overset{*}{\lambda_\varepsilon}(G,Ƶ)]\cup_\varepsilon[(F,T)\overset{*}{\lambda_\varepsilon}(H,M)]$.

Proof: Consider the LHS. Let $(G,Ƶ)\cap_\varepsilon(H,M)=(R,Ƶ\cup M)$, where for all ω∈Ƶ∪M;

$$R(\omega)=\begin{cases} G(\omega) & \omega\in Ƶ\setminus M \\ H(\omega) & \omega\in M\setminus Ƶ \\ G(\omega)\cap H(\omega) & \omega\in Ƶ\cap M \end{cases}$$

Let $(F,T)\overset{*}{\lambda_\varepsilon}(R,Ƶ\cup M)=(N,(T\cup(Ƶ\cup M))$, where for all ω∈T∪(Ƶ∪M);

$$N(\omega)=\begin{cases} F'(\omega) & \omega\in T\setminus(Ƶ\cup M) \\ R'(\omega) & \omega\in(Ƶ\cup M)\setminus T \\ F(\omega)\cup R'(\omega) & \omega\in T\cap(Ƶ\cup M) \end{cases}$$

Thus,

$$N(\omega)=\begin{cases} F'(\omega) & \omega\in T\backslash(Ƶ\cup M)=T\cap Ƶ'\cap M' \\ G'(\omega) & \omega\in(Ƶ\backslash M)\backslash T=T'\cap Ƶ\cap M' \\ H'(\omega) & \omega\in(M\backslash Ƶ)\backslash T=T'\cap Ƶ'\cap M \\ G'(\omega)\cup H'(\omega) & \omega\in(Ƶ\cap M)\backslash T=T'\cap Ƶ\cap M \\ F(\omega)\cup G'(\omega) & \omega\in T\cap(Ƶ\backslash M)=T\cap Ƶ\cap M' \\ F(\omega)\cup H'(\omega) & \omega\in T\cap(M\backslash Ƶ)=T\cap Ƶ'\cap M \\ F(\omega)\cup [G'(\omega)\cup H'(\omega)] & \omega\in T\cap(Ƶ\cap M)=T\cap Ƶ\cap M \end{cases}$$

Now consider the RHS, i.e., $[(F,T)\overset{*}{\lambda_\varepsilon}(G,Ƶ)]\cup_\varepsilon[(F,T)\overset{*}{\lambda_\varepsilon}((H,M)]$. Let $(F,T)\overset{*}{\lambda}(G,Ƶ)=(K,T\cup Ƶ)$. Hence,

for all $\omega\in T\cup Ƶ$;

$$K(\omega)=\begin{cases} F'(\omega) & \omega\in T\backslash Ƶ \\ G'(\omega) & \omega\in Ƶ\backslash T \\ F(\omega)\cup G'(\omega) & \omega\in T\cap Ƶ \end{cases}$$

Let $(F,T)\overset{*}{\lambda_\varepsilon}(H,M)=(S,T\cup M)$, where for all $\omega\in T\cup M$;

$$S(\omega)=\begin{cases} F'(\omega) & \omega\in T\backslash M \\ H'(\omega) & \omega\in M\backslash T \\ F(\omega)\cup H'(\omega) & \omega\in T\cap M \end{cases}$$

Let $(K,T\cup Ƶ)\cup_\varepsilon(S,T\cup M)=(L,(T\cup Ƶ)\cup(T\cup M))$, where for all $\omega\in(T\cup Ƶ)\cup(T\cup M)$;

$$L(\omega)=\begin{cases} K(\omega) & \omega\in(T\cup Ƶ)\backslash(T\cup M) \\ S(\omega) & \omega\in(T\cup M)\backslash(T\cup Ƶ) \\ K(\omega)\cup S(\omega) & \omega\in(T\cup Ƶ)\cap(T\cup M) \end{cases}$$

Thus,

$$L(\omega)=\begin{cases} F'(\omega) & \omega\in(T\setminus Ƶ)\setminus(T\cup M)=\emptyset \\ G'(\omega) & \omega\in(Ƶ\setminus T)\setminus(T\cup M)=T'\cap Ƶ\cap M' \\ F(\omega)\cup G'(\omega) & \omega\in(T\cap Ƶ)\setminus(T\cup M)=\emptyset \\ F'(\omega) & \omega\in(T\setminus M)\setminus(T\cup Ƶ)=\emptyset \\ H'(\omega) & \omega\in(M\setminus T)\setminus(T\cup Ƶ)=T'\cap Ƶ'\cap M \\ F(\omega)\cup H'(\omega) & \omega\in(T\cap M)\setminus(T\cup Ƶ)=\emptyset \\ F'(\omega)\cup F'(\omega) & \omega\in(T\setminus Ƶ)\cap(T\setminus M)=T\cap Ƶ'\cap M' \\ F'(\omega)\cup H'(\omega) & \omega\in(T\setminus Ƶ)\cap(M\setminus T)=\emptyset \\ F'(\omega)\cup [F(\omega)\cup H'(\omega)] & \omega\in(T\setminus Ƶ)\cap(T\cap M)=T\cap Ƶ'\cap M \\ G'(\omega)\cup F'(\omega) & \omega\in(Ƶ\setminus T)\cap(T\setminus M)=\emptyset \\ G'(\omega)\cup H'(\omega) & \omega\in(Ƶ\setminus T)\cap(M\setminus T)= T'\cap Ƶ\cap M \\ G'(\omega)\cup [F(\omega)\cup H'(\omega)] & \omega\in(Ƶ\setminus T)\cap(T\cap M)=\emptyset \\ [F(\omega)\cup G'(\omega)]\cup F'(\omega) & \omega\in(T\cap Ƶ)\cap(T\setminus M)=T\cap Ƶ\cap M' \\ [F(\omega)\cup G'(\omega)]\cup H'(\omega) & \omega\in(T\cap Ƶ)\cap(M\setminus T)=\emptyset \\ [F(\omega)\cup G'(\omega)]\cup [F(\omega)\cup H'(\omega)] & \omega\in(T\cap Ƶ)\cap(T\cap M)=T\cap Ƶ\cap M \end{cases}$$

Therefore,

$$L(\omega)=\begin{cases} G'(\omega) & \omega\in T'\cap Ƶ\cap M' \\ H'(\omega) & \omega\in T'\cap Ƶ'\cap M \\ F'(\omega) & \omega\in T\cap Ƶ'\cap M' \\ U & \omega\in T\cap Ƶ'\cap M \\ G'(\omega)\cup H'(\omega) & \omega\in T'\cap Ƶ\cap M \\ U & \omega\in T\cap Ƶ\cap M' \\ [F(\omega)\cup G'(\omega)]\cup[F(\omega)\cup H'(\omega)] & \omega\in T\cap Ƶ\cap M \end{cases}$$

N=L under the condition $T\cap Ƶ\cap M'=T\cap Ƶ'\cap M=\emptyset$. It is obvious that the condition $T\cap Ƶ'\cap M=T\cap Ƶ\cap M'=\emptyset$ is equivalent to the condition $T\cap(Z\Delta M)=\emptyset$

2) If $T\cap(Z\Delta M)$, then $(F,T) \overset{*}{\lambda}_{\varepsilon}[(G,Ƶ) \cup_{\varepsilon}(H,M)]=[(F,T) \overset{*}{\lambda}_{\varepsilon}(G,Ƶ)]\cap_{\varepsilon}[(F,T) \overset{*}{\lambda}_{\varepsilon}(H,M)]$.

ii) RHS Distributions

1)If $T\cap Ƶ\cap M=\emptyset$, then $[(F,T) \cap_{\varepsilon} (G,Ƶ)] \overset{*}{\lambda}_{\varepsilon}(H,M)= [(F,T) \overset{*}{\lambda}_{\varepsilon}(H,M)] \cup_{\varepsilon} [(G,Ƶ) \overset{*}{\lambda}_{\varepsilon}(H,M)]$.

Proof: Consider the LHS. Let $(F,T)\ \cap_{\varepsilon}(G,Ƶ)=(R,T\cup Ƶ)$, where for all $\omega\in T\cup Ƶ$;

$$R(\omega)=\begin{cases} F(\omega) & \omega\in T\backslash Ƶ \\ G(\omega) & \omega\in Ƶ\backslash T \\ F(\omega)\cap G(\omega) & \omega\in T\cap Ƶ \end{cases}$$

Let $(R,T\cup Ƶ)\ \overset{*}{\lambda_{\varepsilon}}(H,M)=(N,(T\cup Ƶ)\cup M)$, where for all $\omega\in(T\cup Ƶ)\cup M$;

$$N(\omega)=\begin{cases} R'(\omega) & \omega\in(T\cup Ƶ)\backslash M \\ H'(\omega) & \omega\in M\backslash(T\cup Ƶ) \\ R(\omega)\cup H'(\omega) & \omega\in(T\cup Ƶ)\cap M \end{cases}$$

Thus,

$$N(\omega)=\begin{cases} F'(\omega) & \omega\in(T\backslash Ƶ)\backslash M=T\cap Ƶ'\cap M' \\ G'(\omega) & \omega\in(Ƶ\backslash T)\backslash M=T'\cap Ƶ\cap M' \\ F'(\omega)\cup G'(\omega) & \omega\in(T\cap Ƶ)\backslash M=T\cap Ƶ\cap M' \\ H'(\omega) & \omega\in M\backslash(T\cup Ƶ)=T'\cap Ƶ'\cap M \\ F(\omega)\cup H'(\omega) & \omega\in(T\backslash Ƶ)\cap M=T\cap Ƶ'\cap M \\ G(\omega)\cup H'(\omega) & \omega\in(Ƶ\backslash T)\cap M=T'\cap Ƶ\cap M \\ [F(\omega)\cap G(\omega)]\cup H'(\omega) & \omega\in(T\cap Ƶ)\cap M=T\cap Ƶ\cap M \end{cases}$$

Consider the RHS, i.e., $[(F,T)\ \overset{*}{\lambda_{\varepsilon}}(H,M)]\cup_{\varepsilon}[(G,Ƶ)\ \overset{*}{\lambda_{\varepsilon}}(H,M)]$. Let $(F,T)\overset{*}{\lambda_{\varepsilon}}(H,M)=(K,T\cup Ƶ)$, where for all $\omega\in T\cup M$;

$$K(\omega)=\begin{cases} F'(\omega) & \omega\in T\backslash M \\ H'(\omega) & \omega\in M\backslash T \\ F(\omega)\cup H'(\omega) & \omega\in T\cap M \end{cases}$$

Let $(G,Ƶ)\ \overset{*}{\lambda_{\varepsilon}}(H,M)=(S,T\cup M)$, where for all $\omega\in Ƶ\cup M$;

$$S(\omega)=\begin{cases} G'(\omega) & \omega\in Ƶ\backslash M \\ H'(\omega) & \omega\in M\backslash Ƶ \\ G(\omega)\cup H'(\omega) & \omega\in Ƶ\cap M \end{cases}$$

Let $(K,T\cup M)\ \cup_{\varepsilon}(S,Ƶ\cup M)=(L,(T\cup M)\cup(Ƶ\cup M))$, where for all $\omega\in(T\cup M)\cup(Ƶ\cup M)$;

$$L(\omega)=\begin{cases} K(\omega) & \omega\in(T\cup M)\backslash(Ƶ\cup M) \\ S(\omega) & \omega\in(Ƶ\cup M)\backslash(T\cup M) \\ K(\omega)\cup S(\omega) & \omega\in(T\cup M)\cap(Ƶ\cup M) \end{cases}$$

Thus,

$$
L(\omega)=\begin{cases}
F'(\omega) & \omega\in(T\backslash M)\backslash(Ƶ\cup M)=T\cap Ƶ'\cap M' \\
H'(\omega) & \omega\in(M\backslash T)\backslash(Ƶ\cup M)=\emptyset \\
F(\omega)\cup H'(\omega) & \omega\in(T\cap M)\backslash(Ƶ\cup M)=\emptyset \\
G'(\omega) & \omega\in(Ƶ\backslash M)\backslash(T\cup M)=T'\cap Ƶ\cap M' \\
H(\omega) & \omega\in(M\backslash Ƶ)\backslash(T\cup M)=\emptyset \\
G(\omega)\cup H'(\omega) & \omega\in(Ƶ\cap M)\backslash(T\cup M)=\emptyset \\
F'(\omega)\cup G'(\omega) & \omega\in(T\backslash M)\cap(Ƶ\backslash M)=T\cap Ƶ\cap M' \\
F'(\omega)\cup H'(\omega) & \omega\in(T\backslash M)\cap(M\backslash Ƶ)=\emptyset \\
F'(\omega)\cup [G(\omega)\cup H'(\omega)] & \omega\in(T\backslash M)\cap(Ƶ\cap M)=\emptyset \\
H'(\omega)\cup G'(\omega) & \omega\in(M\backslash T)\cap(Ƶ\backslash M)=\emptyset \\
H'(\omega)\cup H'(\omega) & \omega\in(M\backslash T)\cap(M\backslash Ƶ)= T'\cap Ƶ'\cap M \\
H'(\omega)\cup [G(\omega)\cup H'(\omega)] & \omega\in(M\backslash T)\cap(Ƶ\cap M)=T'\cap Ƶ\cap M \\
[F(\omega)\cup H'(\omega)]\cup G'(\omega) & \omega\in(T\cap M)\cap(Ƶ\backslash M)=\emptyset \\
[F(\omega)\cup H'(\omega)]\cup H'(\omega) & \omega\in(T\cap M)\cap(M\backslash Ƶ)=T\cap Ƶ'\cap M \\
[F(\omega)\cup H'(\omega)]\cup [G(\omega)\cup H'(\omega)] & \omega\in(T\cap M)\cap(Ƶ\cap M)=T\cap Ƶ\cap M
\end{cases}
$$

Hence,

$$
L(\omega)=\begin{cases}
F'(\omega) & \omega\in T\cap Ƶ'\cap M' \\
G'(\omega) & \omega\in T'\cap Ƶ\cap M' \\
F'(\omega)\cup G'(\omega) & \omega\in T\cap Ƶ\cap M' \\
H'(\omega) & \omega\in T'\cap Ƶ'\cap M \\
G(\omega)\cup H'(\omega) & \omega\in T'\cap Ƶ\cap M \\
F(\omega)\cup H'(\omega) & \omega\in T\cap Ƶ'\cap M \\
[F(\omega)\cup H'(\omega)]\cup [G(\omega)\cup H'(\omega)] & \omega\in T\cap Ƶ\cap M
\end{cases}
$$

It is observed that N=L under the condition $T\cap Ƶ\cap M'=\emptyset$.

2)If $T\cap Ƶ\cap M'=\emptyset$, then $[(F,T)\cup_{\varepsilon}(G,Ƶ)]\overset{*}{\lambda_{\varepsilon}}(H,M)=[(F,T)\overset{*}{\lambda_{\varepsilon}}(H,M)]\cup_{\varepsilon}[(G,Ƶ)\overset{*}{\lambda_{\varepsilon}}(H,M)]$.

Theorem 4.3. Let $(F,T),(G,Ƶ),(H,M)$ be soft sets over U. The following distributions of the complementary extended lambda operation over soft binary piecewise operations hold:

i) LHS Distributions

1) If $T\cap(Z\Delta M)=\emptyset$, then $(F,T)\overset{*}{\lambda_{\varepsilon}}[(G,Ƶ)\,\tilde{\cap}\,(H,M)] = [(F,T)\overset{*}{\lambda_{\varepsilon}}(G,Ƶ)]\,\tilde{\cup}\,[(F,T)\overset{*}{\lambda_{\varepsilon}}(H,M)]$.

Proof: Consider the LHS. Let $(G,Ƶ)\,\tilde{\cap}\,(H,M)=(R,Ƶ)$, where for all $\omega\in Z$;

$$R(\omega)=\begin{cases} G(\omega) & \omega\in Ƶ\setminus M \\ G(\omega)\cap H(\omega) & \omega\in Ƶ\cap M \end{cases}$$

Let $(F,T)\overset{*}{\lambda_{\varepsilon}}(R,Ƶ)=(N,T\cupƵ)$, where for all $\omega\in T\cupƵ$;

$$N(\omega)=\begin{cases} F'(\omega) & \omega\in T\setminus Ƶ \\ R'(\omega) & \omega\in Ƶ\setminus T \\ F(\omega)\cup R'(\omega) & \omega\in T\cap Ƶ \end{cases}$$

Hence,

$$N(\omega)=\begin{cases} F'(\omega) & \omega\in T\setminus Ƶ \\ G'(\omega) & \omega\in(Ƶ\setminus M)\setminus T=T'\cap Ƶ\cap M' \\ G'(\omega)\cup H'(\omega) & \omega\in(Ƶ\cap M)\setminus T=T'\cap Ƶ\cap M \\ F(\omega)\cup G'(\omega) & \omega\in T\cap(Ƶ\setminus M)=T\cap Ƶ\cap M' \\ F(\omega)\cup[G'(\omega)\cup H'(\omega)] & \omega\in T\cap(Ƶ\cap M)=T\cap Ƶ\cap M \end{cases}$$

Consider the RHS, i.e., $[(F,T)\overset{*}{\lambda_{\varepsilon}}(G,Ƶ)]\,\tilde{\cup}\,[(F,T)\overset{*}{\lambda_{\varepsilon}}(H,M)]$. Let $(F,T)\overset{*}{\lambda_{\varepsilon}}(G,Ƶ)=(K,T\cupƵ)$, where for all $\omega\in T\cupƵ$;

$$K(\omega)=\begin{cases} F'(\omega) & \omega\in T\setminus Ƶ \\ G'(\omega) & \omega\in Ƶ\setminus T \\ F(\omega)\cup G'(\omega) & \omega\in T\cap Ƶ \end{cases}$$

Let $(F,T)\overset{*}{\lambda_{\varepsilon}}(H,M)=(S,T\cup M)$, where for all $\omega\in T\cup M$;

$$S(\omega)=\begin{cases} F'(\omega) & \omega\in T\setminus M \\ H'(\omega) & \omega\in M\setminus T \\ F(\omega)\cup H'(\omega) & \omega\in T\cap M \end{cases}$$

Let $(K,T\cupƵ)\,\tilde{\cup}\,(S,T\cup M)=(L,(T\cupƵ)\cup(T\cup M))$, where for all $\omega\in(T\cupƵ)\cup(T\cup M)$;

$$L(\omega)=\begin{cases} K(\omega) & \omega\in(T\cupƵ)\setminus(T\cup M) \\ K(\omega)\cup S(\omega) & \omega\in(T\cupƵ)\cap(T\cup M) \end{cases}$$

Thus,

$$L(\omega)=\begin{cases} F'(\omega) & \omega\in(T\setminus Ƶ)\setminus(T\cup M)=\emptyset \\ G'(\omega) & \omega\in(Ƶ\setminus T)\setminus(T\cup M)=T'\cap Ƶ\cap M' \\ F(\omega)\cup G'(\omega) & \omega\in(T\cap Ƶ)\setminus(T\cup M)=\emptyset \\ F'(\omega)\cup F'(\omega) & \omega\in(T\setminus Ƶ)\cap(T\setminus M)=T\cap Ƶ'\cap M' \\ F'(\omega)\cup H'(\omega) & \omega\in(T\setminus Ƶ)\cap(M\setminus T)=\emptyset \\ F'(\omega)\cup [F(\omega)\cup H'(\omega)] & \omega\in(T\setminus Ƶ)\cap(T\cap M)=T\cap Ƶ'\cap M \\ G'(\omega)\cup F'(\omega) & \omega\in(Ƶ\setminus T)\cap(T\setminus M)=\emptyset \\ G'(\omega)\cup H'(\omega) & \omega\in(Ƶ\setminus T)\cap(M\setminus T)= T'\cap Ƶ\cap M \\ G'(\omega)\cup [F(\omega)\cup H'(\omega)] & \omega\in(Ƶ\setminus T)\cap(T\cap M)=\emptyset \\ [F(\omega)\cup G'(\omega)]\cup F'(\omega) & \omega\in(T\cap Ƶ)\cap(T\setminus M)=T\cap Ƶ\cap M' \\ [F(\omega)\cup G'(\omega)]\cup H'(\omega) & \omega\in(T\cap Ƶ)\cap(M\setminus T)=\emptyset \\ [F(\omega)\cup G'(\omega)]\cup [F(\omega)\cup H'(\omega)] & \omega\in(T\cap Ƶ)\cap(T\cap M)=T\cap Ƶ\cap M \end{cases}$$

Hence,

$$L(\omega)=\begin{cases} G'(\omega) & \omega\in T'\cap Ƶ\cap M' \\ F'(\omega) & \omega\in T\cap Ƶ'\cap M' \\ U & \omega\in T\cap Ƶ'\cap M \\ G'(\omega)\cup H'(\omega) & \omega\in T'\cap Ƶ\cap M \\ U & \omega\in T\cap Ƶ\cap M' \\ [F(\omega)\cup G'(\omega)]\cup [F(\omega)\cup H'(\omega)] & \omega\in T\cap Ƶ\cap M \end{cases}$$

Here, if we consider $T\setminus Ƶ$ in the function N, since $T\setminus Ƶ=T\cap Ƶ'$, if an element is in the complement of Z, then the element is either in $M\setminus Ƶ$ or in $(M\cup Ƶ)'$. Thus, if $\omega\in T\setminus Ƶ$, then $\omega\in T\cap M\cap Ƶ'$ veya $\omega\in T\cap M'\cap Ƶ'$. Therefore, N=L under the $T\cap Ƶ'\cap M=T\cap Ƶ\cap M'=\emptyset$ It is obvious that the condition $T\cap Ƶ'\cap M=T\cap Ƶ\cap M'=\emptyset$ is equivalent to the condition $T\cap(Z\Delta M)=\emptyset$.

2) If $T\cap(Z\Delta M)=\emptyset$, then $(F,T) \overset{*}{\lambda_{\varepsilon}}[(G,Ƶ) \,\tilde{\cup}\, (H,M)] = [(F,T) \overset{*}{\lambda_{\varepsilon}}(G,Ƶ)] \,\tilde{\cap}\, [(F,T) \overset{*}{\lambda_{\varepsilon}}(H,M)]$.

ii) RHS Distributions

1) If $T\cap(Z\Delta M)=\emptyset$, then $[(F,T) \,\tilde{\cup}\, (G,Ƶ)] \overset{*}{\lambda_{\varepsilon}} (H,M) = [(F,T) \overset{*}{\lambda_{\varepsilon}}(H,M)] \,\tilde{\cup}\, [(G,Ƶ) \overset{*}{\lambda_{\varepsilon}} (H,M)]$

Proof: Consider the LHS. Let (F,T) $\tilde{\cup}$ (G,Ƶ)=(R,T), where for all ω∈T;

$$R(\omega)=\begin{cases} F(\omega) & \omega\in T\setminus Ƶ \\ F(\omega)\cup G(\omega) & \omega\in T\cap Ƶ \end{cases}$$

Let (R,T) $\overset{*}{\lambda_\varepsilon}$(H,M) =(N,T∪M), where for all ω∈T∪M;

$$N(\omega)=\begin{cases} R'(\omega) & \omega\in T\setminus M \\ H'(\omega) & \omega\in M\setminus T \\ R(\omega)\cup H'(\omega) & \omega\in T\cap M \end{cases}$$

Thus,

$$N(\omega)=\begin{cases} F'(\omega) & \omega\in(T\setminus Ƶ)\setminus M=T\cap Ƶ'\cap M' \\ F'(\omega)\cap G'(\omega) & \omega\in(T\cap Ƶ)\setminus M=T\cap Ƶ\cap M' \\ H'(\omega) & \omega\in M\setminus T \\ F(\omega)\cup H'(\omega) & \omega\in(T\setminus Ƶ)\cap M=T\cap Ƶ'\cap M \\ [F(\omega)\cup G(\omega)]\cup H'(\omega) & \omega\in(T\cap Ƶ)\cap M=T\cap Ƶ\cap M \end{cases}$$

Now consider the RHS, i.e., [(F,T) $\overset{*}{\lambda_\varepsilon}$(H,M)] $\tilde{\cup}$ [(G,Ƶ) $\overset{*}{\lambda_\varepsilon}$(H,M)]. Let (F,T) $\overset{*}{\lambda_\varepsilon}$(H,M)=(K,T∪M), where for all ω∈T∪M

$$K(\omega)=\begin{cases} F'(\omega) & \omega\in T\setminus M \\ H'(\omega) & \omega\in M\setminus T \\ F(\omega)\cup H'(\omega) & \omega\in T\cap M \end{cases}$$

Let (G,Ƶ) $\overset{*}{\lambda_\varepsilon}$ (H,M)=(S, Ƶ∪M), where for all ω∈Ƶ∪M;

$$S(\omega)=\begin{cases} G'(\omega) & \omega\in Ƶ\setminus M \\ H'(\omega) & \omega\in M\setminus Ƶ \\ G(\omega)\cup H'(\omega & \omega\in Ƶ\cap M \end{cases}$$

Let (K,T∪M) $\tilde{\cup}$ (S,Ƶ∪M)=(L,(T∪M)∪(Ƶ∪M)), where for all ω∈(T∪M)∪(Ƶ∪M);

$$L(\omega)=\begin{cases} K(\omega) & \omega\in(T\cup M)\setminus(Ƶ\cup M) \\ K(\omega)\cup S(\omega) & \omega\in(T\cup M)\cap(Ƶ\cup M) \end{cases}$$

Thus,

$$L(\omega)=\begin{cases} F'(\omega) & \omega\in(T\setminus M)\setminus(Ƶ\cup M)=T\cap Ƶ'\cap M' \\ H'(\omega) & \omega\in(M\setminus T)\setminus(Ƶ\cup M)=\emptyset \\ F(\omega)\cup H'(\omega) & \omega\in(T\cap M)\setminus(Ƶ\cup M)=\emptyset \\ F'(\omega)\cup G'(\omega) & \omega\in(T\setminus M)\cap(Ƶ\setminus M)=T\cap Ƶ\cap M' \\ F'(\omega)\cup H'(\omega) & \omega\in(T\setminus M)\cap(M\setminus Ƶ)=\emptyset \\ F'(\omega)\cup [G(\omega)\cup H'(\omega)] & \omega\in(T\setminus M)\cap(Ƶ\cap M)=\emptyset \\ H'(\omega)\cup G'(\omega) & \omega\in(M\setminus T)\cap(Ƶ\setminus M)=\emptyset \\ H'(\omega)\cup H'(\omega) & \omega\in(M\setminus T)\cap(M\setminus Ƶ)= T'\cap Ƶ'\cap M \\ H'(\omega)\cup [G(\omega)\cup H'(\omega)] & \omega\in(M\setminus T)\cap(Ƶ\cap M)=T'\cap Ƶ\cap M \\ [F(\omega)\cup H'(\omega)]\cup G'(\omega) & \omega\in(T\cap M)\cap(Ƶ\setminus M)=\emptyset \\ [F(\omega)\cup H'(\omega)]\cup H'(\omega) & \omega\in(T\cap M)\cap(M\setminus Ƶ)=T\cap Ƶ'\cap M \\ [F(\omega)\cup H'(\omega)]\cup [G(\omega)\cup H'(\omega)] & \omega\in(T\cap M)\cap(Ƶ\cap M)=T\cap Ƶ\cap M \end{cases}$$

Hence,

$$L(\omega)=\begin{cases} F'(\omega) & \omega\in T\cap Ƶ'\cap M' \\ F'(\omega)\cup G'(\omega) & \omega\in T\cap Ƶ\cap M' \\ H'(\omega) & \omega\in T'\cap Ƶ'\cap M \\ G(\omega)\cup H'(\omega) & \omega\in T'\cap Ƶ\cap M \\ F(\omega)\cup H'(\omega) & \omega\in T\cap Ƶ'\cap M \\ [F(\omega)\cup H'(\omega)]\cup [G(\omega)\cup H'(\omega)] & \omega\in T\cap Ƶ\cap M \end{cases}$$

Here, if we consider $M\setminus T$ in the function N, since $M\setminus T=M\cap T'$, then if an element is in the complement of T, it is either in $Ƶ\setminus T$ or in $(Ƶ\cup T)'$. From here, N=L under $T'\cap Ƶ\cap M=T\cap Ƶ\cap M'=\emptyset$. It is obvious that the condition $T'\cap Ƶ\cap M=T\cap Ƶ\cap M'=\emptyset$ is equivalent to the condition $(T\Delta M)\cap Ƶ=\emptyset$.

2) If $T\cap(Z\Delta M)=\emptyset$, then $[(F,T)\tilde{\cap}(G,Ƶ)]\overset{*}{\lambda_{\varepsilon}}(H,M)=[(F,T)\overset{*}{\lambda_{\varepsilon}}(H,M)]\tilde{\cap}[(G,Ƶ)\overset{*}{\lambda_{\varepsilon}}(H,M)]$

5. Conclusion

Soft set operations are the most fundamental building block of soft set theory for the progress of soft set theory in both theoretical and practical fields. Since the theory was introduced in 1999, many restricted and extended operations have been introduced. This work proposes and investigates the algebraic properties of a novel soft set operation, called complementary extended lambda operation. We deal with the distributions of complementary extended lambda operations over other types of soft set operations. Since the concepts linked to soft set operations are just as important for soft sets as basic operations from classical set theory and thus examining the algebraic structures of soft sets in connection to new soft set operations offers us a thorough knowledge of their application as well as new examples of algebraic structures, we believe that this work contributes to the literature of both classical algebra and soft set theory. To determine what algebraic structures are produced in the classes of soft sets with a fixed parameter set or over the universe, future studies may look at different types of complementary extended soft set operations along with their distributions and properties. Moreover, this novel operation can be conveyed to bipolar soft sets, lattice ordered soft sets and double framed soft sets and the researchers may explore which algebraic structures are formed with this operation when combined with other operations, and new decision-making methods may be proposed with the inspiration of the operation.

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Author's Contribution

The contributions of the authors are equal.

Conflict of Interest

The authors have declared that there is no conflict of interest.

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